Credit Risk

Lecture 3 – Credit Risk ratings and Introduction to Climate Risk

Loïc BRIN

École Nationale des Ponts et Chaussées
Département Ingénieurie Mathématique et Informatique (IMI) – Master II

Objectives of the lecture

Teaching objectives

At the end of this lecture, you will:

- understand **credit ratings**;
- understand how basic models such as **logistic regressions** can be calibrated;
- get introduced to credit and **climate risk modeling**.

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The various conceptions of default

Is default a binary concept?

- For an **accountant**: it is possible to book losses even though the counterparty has made all its payments until now (cf. IAS 39 / IFRS 9, see Lecture 5);
- For the **regulator**, according to Basel Committee: "A default is considered to have occurred with regard to a particular obligor when one or more of the following events has taken place:
  - It is determined that the obligor is **unlikely to pay** its debt obligations (principal, interest, or fees) in full;
  - A **credit loss event** associated with any obligation of the obligor, such as a charge-off, specific provision, or distressed restructuring involving the forgiveness or postponement of principal, interest, or fees;
  - The obligor is past due **more than 90 days** on any credit obligation;
  - The obligor has **filed for bankruptcy** or similar protection from creditors."[1]
- For the market and rating agencies:
  - **bankruptcy**, liquidation or stop of activity;
  - **unpaid flow**;
  - **restructuring**.

[1] Basel Committee on Banking Supervision, The Internal Rating Based Approach.

Ratings and rating agencies

What is the role of rating agencies?

Rating agencies

Rating agencies give grades to economic agents that reflect their **ability to reimburse** borrowed money, thanks to **qualitative and quantitative** criteria gathered by their analysts. These critiera can be:

- expected future cash flows;
- short term, long term liabilities;
- structure of the liabilities;
- countries of activity;
- competition in the market;
- quality of the management.

The most famous rating agencies are **Moody's**, **Fitch Ratings** and **Standard and Poors**.

Ratings and rating agencies

What does a rating scale look like?

Rating scale

The grades are the following (for S&P):

Investment Grades	Speculative Grades
AAA	BB+
AA+	BB
AA	BB-
AA-	B+
A+	B
A	B-
A-	CCC+
BBB+	CCC
BBB	CCC-
BBB-	CC
	C

Ratings and rating agencies

How ratings relate to default?

It appears from the historical data that there is **a strong exponential relationship** between the rating of a firm and its one-year probability of default. That is to say the probability that the firm will default within the next year.

We have: $\text{PD} = 6 \times 10^{-6} \times e^{0.64 * \text{Rating}}$.

Ratings of well-known firms

Firm	S&P rating
SG	A
BNP	A+
Total	A+
EDF	BBB
Accor	BBB-

Transition matrix

How do ratings migrate?

Definition of Transition Matrix

In credit risk, **a transition matrix**, $\boldsymbol{M}_{t,t+1} = (m_{ij})_{ij}$, is a matrix where:

$$m_{ij} = \mathbb{P}(\text{Grade}_{t+1} = j \mid \text{Grade}_t = i)$$

Where i and j are the grades presented earlier.

S&P's transition matrix – From 1981 to 2013

(in %)	AAA	AA	A	BBB	BB	B	CCC	D	NR[2]
AAA	87.11	8.88	0.53	0.05	0.08	0.03	0.05	0	3.27
AA	0.55	86.39	8.26	0.56	0.06	0.07	0.02	0.02	4.07
A	0.03	1.87	87.34	5.48	0.35	0.14	0.02	0.07	4.7
BBB	0.01	0.12	3.59	85.22	3.82	0.59	0.13	0.21	6.31
BB	0.02	0.04	0.15	5.2	76.28	7.08	0.69	0.8	9.74
B	0	0.03	0.11	0.22	5.48	73.89	4.46	4.11	11.7
CCC	0	0	0.15	0.23	0.69	13.49	43.81	26.87	14.76

[2] "Non-rated" or "Unrated". Some "companies" may decide to stop being rated by a given rating agency.

Transition matrix

What properties should be expected from a transition matrix?

Several properties of transition matrices

Among the **properties of the transition matrices**, note that:

- Each row sums to 1;
- They are dominant;
- In the case of homogeneity we have that: $\boldsymbol{M}_{t,t+n} = \boldsymbol{M}^{n}_{t,t+1}$.

The generator for homogeneous Markov chains

The **generator for a Markov chain** $(\boldsymbol{M}_{t,t+n})_n$ is the matrix $\boldsymbol{Q}$ so that:

$$\forall (t, T), \quad \boldsymbol{M}_{t,T} = \exp\left((T - t)\boldsymbol{Q}\right) \qquad \text{with } \exp(\boldsymbol{A}) = \sum_{n \geq 0} \frac{\boldsymbol{A}^n}{n!}$$

Would such a matrix exist (see [Israel et al., 2001]), we have:

$$\boldsymbol{Q} = \sum_{n > 0} (-1)^{n-1} \frac{(\boldsymbol{M}_{t,t+1} - \boldsymbol{I})^n}{n}$$

Transition matrix

How to estimate transition matrices?

Two techniques to estimate transition matrices

There are two techniques to estimate the generator of transition matrices:

- By **cohorts**: it consists in computing the average number of agents that change from rating i to j within one year, for all (i, j);
- By **durations**: it consists in looking for instantaneous probability, for an agent, of changing from rating i to j. The likelihood of changing of rating, from i to j, in t, is:

$$e^{\lambda_{ii} t} \lambda_{ij}$$

By maximizing the likelihood of these transitions, one can estimate $(\lambda_{ij})_{ij}$.

Transition matrix

Do the maths fit reality?

Transition matrices are not Markov matrices

The markovian assumption is in **contradiction** with phenomena described by the data.

For example, a firm which has recently experienced a downgrade to rating j, is more likely to experience another one, as opposed to a firm that has had rating j for a long time.

Quiz

Conclusion

Default and rating-based models

- Default can have **various definitions depending on the context**;
- Rating agencies provide **ratings that can be used to estimate the one-year probability of default of firms**;
- A transition matrix that allow to **describe ratings migrations**.

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What do we want to predict?

Can we predict default?

- Will a firm/customer **default within a given period of time** (year, month, etc.)?

$$Y = \begin{cases} 1 & \text{if default within a given period,} \\ 0 & \text{otherwise.} \end{cases}$$

- Or better yet: What is the probability that a firm/customer will default within a given period of time, given the information we have?

$$p(X) = \mathbb{P}(Y = 1|X)$$

- For a firm: X can be financial data, market data, country, activity sector, etc.
- For a customer: X can be age, job situation, salary, debt level, etc.

Theoretical framework of the logistic regression

The logistic regression's assumptions

Logistic regression's model

The **logistic regression model** can be defined the following ways:

$$p(X) = \mathbb{P}(Y = 1|X) = \frac{e^{\beta_0+\beta_1 X_1+\ldots+\beta_p X_p}}{1 + e^{\beta_0+\beta_1 X_1+\ldots+\beta_p X_p}}$$

or equivalently

$$\ln\left(\frac{p(X)}{1 - p(X)}\right) = \beta_0 + \beta_1 X_1 + \ldots + \beta_p X_p \tag{1}$$

where $X = (X_1, \ldots, X_p)$

No closed-form solution

In practice, the vector of parameters $(\beta_0, \ldots, \beta_p)$ is estimated by maximizing the likelihood. Contrary to linear regression, there is **no closed-form solution**, which can therefore lead to different estimations depending on the algorithm/software chosen.

Theoretical framework of the logistic regression

How to interpret the coefficients? (I/II)

Logistic regression's coefficients can be interpreted through the concept of **odds ratio** using equation (1).

Coefficient interpretation of a logistic regression with one binary predictor (I/II)

We consider the following logistic modelling where the default ($Y = 1$) only depends on being a student ($X = 1$) or not ($X = 0$):

$$\ln\left(\frac{p(X)}{1 - p(X)}\right) = \beta_0 + \beta_1 X$$

Then $\exp(\beta_0)$ is the odds ratio of a non-student being in the honor class (default):

$$\exp(\beta_0) = \frac{\mathbb{P}(Y = 1|X = 0)}{1 - \mathbb{P}(Y = 1|X = 0)} = \frac{\mathbb{P}(\text{default}|\text{non-student})}{\mathbb{P}(\text{no default}|\text{non-student})}$$

$\exp(\beta_1)$ is the ratio of the odds for student to the odds for non-student:

$$\exp(\beta_1) = \frac{\mathbb{P}(Y = 1|X = 1)}{1 - \mathbb{P}(Y = 1|X = 1)} \Big/ \frac{\mathbb{P}(Y = 1|X = 0)}{1 - \mathbb{P}(Y = 1|X = 0)}$$

Theoretical framework of the logistic regression

How to interpret the coefficients? (II/II)

Coefficient interpretation of a logistic regression with one binary predictor (II/II)

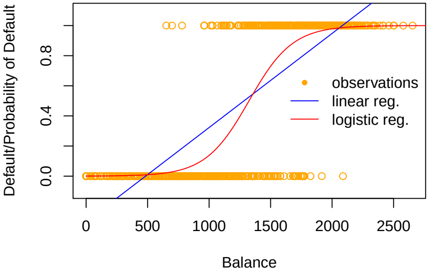
- We deduce from the previous slide that according to the model a non-student is $\exp(\beta_0)$ time(s) as likely to default as to not.
- Let's assume that $\beta_1 > 0$. The odds for a student is $\exp(\beta_1) - 1$ times higher than the odds for a non-student.

To learn more about logistic regression coefficients' interpretation:

Tutorial

Linear vs logistic regression

Why not use linear regression?



Source: The Default dataset from [Gareth et al., 2009]

The main reason for using logistic regression instead of linear regression is that the predictions are inside the $[0, 1]$ interval, making them easier to **interpret as probabilities**.

Conclusion

Why and why not use logistic regression to model default?

Pros:

- Can be **easily interpreted**:
  - reducing the risk of modeling errors;
  - making it easier to be audited (validation team, regulator, etc.).
- Provides an **estimation of the probability of default**;
- Can be converted into a **score card** to facilitate the use of the model.

Cons:

- Lacks prediction power (logistic regression **cannot model non-linear relationships**);
- In practice the continuous predictors must be binned manually;
- Requires variables selection/regularization (lasso).

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The tragedy of the horizon

There is a growing international consensus that **climate change is unequivocal**. This change raises **new risks** for the banking system and financial stability:

- **Physical risks** (e.g. flood, exceptional droughts - insurance natural events losses tripled since 90s) and
- **Transition risks** (e.g. strengthening of carbon tax or environmental requirements such as petrol ban) may drastically affect banking portfolios and turn out to be a credible source of systemic risk and require therefore attention from central banks, in their financial stability mandate.

Furthermore, in line with Paris Accords (2015), there is a **need to support green investment** and favor the transition to a low carbon economy.

The tragedy of the horizon

The tragedy of the horizon

See Mark Carney: Speech at Lloyd's of London, 2015 (Breaking the tragedy of the horizon – climate change and financial stability):

- **physical impacts** of climate change will be felt over a **long-term horizon**, with massive costs and possible civilisational impacts on future generations vs
- the **time horizon in which financial**, economic and political players plan and act is much shorter

YouTube

Implications for banks

Which impact on real economy

- Extreme natural events
- Population Moves
- Black Swans/Green Swans
- Changes in consumption
- Reputation
- Political responses
- Stranded assets
- Changes in business models

Climate risk (impact of carbon prices)and credit risk

Credit model

Link between CO2 emissions and global temperature increase

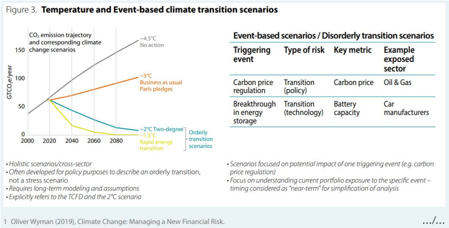


Figure 3. **Temperature and Event-based climate transition scenarios**

Event-based scenarios / Disorderly transition scenarios

Triggering event	Type of risk	Key metric	Example exposed sector
Carbon price regulation	Transition (policy)	Carbon price	Oil & Gas
Breakthrough in energy storage	Transition (technology)	Battery capacity	Car manufacturers

- *Holistic scenarios/cross-sector*
- *Often developed for policy purposes to describe an orderly transition, not a stress scenario*
- *Requires long-term modeling and assumptions*
- *Explicitly refers to the TCFD and the 2°C scenario*

- *Scenarios focused on potential impact of one triggering event (e.g. carbon price regulation)*
- *Focus on understanding current portfolio exposure to the specific event – timing considered as "near-term" for simplification of analysis*

1 Oliver Wyman (2019), Climate Change: Managing a New Financial Risk. .../...

Source: Extract from "Overview of Environmental Risk Analysis by Financial Institutions", Technical Report from NGFS

Solution

- Carbon tax;
- Impact on firms EBITDA (hence profitability);
- impact on firm credit risk.

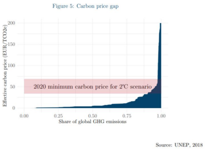
Link between Asset Value and EBITDA

The Asset Value of the firm is equal to the expected sum of discounted future cash flows (Modigliani-Miller)

$$Asset \equiv Cashflows(EBITDA - tbd/simplification)$$

Bouchet/Le Guenedal idea ("Credit Risk Sensitivity to Carbon Price", Working Paper 95-2019, Amundi, 2020):

- assessing the impact of carbon tax on EBITDA (through several scenarios)
  - collect all firms cash flows
  - collect all firms CO2 emissions and compute hypothetical carbon tax
  - deduce loss in cashflow from additional CO2 tax
- converting this impact in a proportional impact (w.r.t. cashflow level)
- apply same impact on Asset Value
- through Merton model, assess the impact on credit risk valuation (debt value, default probability)



Source: Extract from Bouchet/Le Guenedal

Computing probability of default

Observable variables are:

- Equity and Total Debt in current scenario/situation
- equity volatility
- assess average Debt Duration T

Then solve ($Assets$ and σ_{Assets}) :

$$Equity = Assets\ \Phi(d_1) - Debt\ e^{-rT}\Phi(d_2) \tag{2}$$

$$Equity = \frac{\sigma_{Assets}}{\sigma_{Equity}}\Phi(d_1)\ Assets \tag{3}$$

recalling that d_1 and d_2 are deduced from $Assets$ and σ_{Assets} and observable variables :

$$d_1 = \frac{\ln\left(\frac{Assets}{Debt}\right) + (r + \frac{\sigma_V^2}{2}) \times T}{\sigma_V\sqrt{T}} \tag{4}$$

$$d_2 = d_1 - \sigma_V\sqrt{T} \tag{5}$$

Bouchet/Le Guenedal results/use

Extracts from Bouchet/Le Guenedal:

Figure 7: Scenario selection and global distribution of carbon price (all models and scenarios)

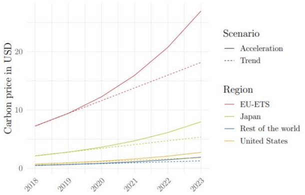


Figure 8: Medium-term impact on EBITDA

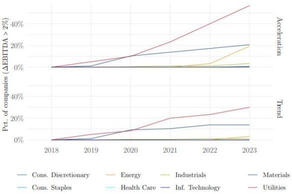
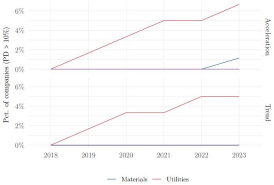


Figure 9: Medium-term impact on probabilities of default



References I

- Bouchet and Le Guenedal (2020).
  Credit Risk Sensitivity to Carbon Price.
  SSRN/Amundi Working Paper.
  Link.

- Brunel and Roger (2015).
  Le Risque de Crédit : des modèles au pilotage de la banque.
  Economica.
  Link.

- Gareth et al. (2009).
  An Introduction to Statistical Learning.
  Springer.
  Link.

- Israel et al. (2001).
  Finding Generators for Markov Chains via Empirical Transition Matrices.
  Mathematical Finance.
  Link.