Credit Risk Cheat Sheet

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Abstract

This short document lists the main formulas, concepts and definitions of the class. Framed definitions starting with ► are the key concepts of the class that must be known. ★ are important information to keep in mind as general knowledge. ▲ refers to traps and points of attention.

BOND VALUATION

Lecture 1

► **Price of a bond.** The price of a bond serving fixed coupons C (fixed rate bond) in $(t_1, ..., t_n)$, which maturity is T and nominal N is:

$$\bar{B}^A(0,T) = \sum_{i=1}^{n} \frac{C}{(1+r_i^A)^{t_i}} + \frac{N}{(1+r_T^A)^T}$$

where:

$$r_i^A = r_i + s^A$$

with r_i, the risk-free rate in i, and s^A, the so-called "Z-spread" or more commonly, the spread of the counterparty A.

▲ **What is a risk-free rate?.** the risk-free rate is usually considered as the Constant Maturity Swap (CMS) price, for different maturities. For example, in Europe, the 10 years, risk-free rate, would be the CMS 10y that exchanges a fixed rate with EURIBOR 3M (ticker BBG being EUSA10Y).

► **Price of a bond – continuous rate and coupon.** Considering a continuous coupon, the formula for a bond of nominal 1, is:

$$\bar{B}^A(0,T) = 1 + (c - r^A)\frac{1 - e^{-r^A T}}{r^A}$$

(only if we consider r_i and s^A as constant)

▲ **A chicken and egg problem.** Note that this is a chicken and egg problem: the spread is extracted from the price and the price is deduced from the spread. In practice, the market, by buying and selling bonds, agrees to a price from which one can extract a spread to price new bonds or credit derivatives.

► **The implied probability of default.** The no-arbitrage assumption gives us:

$$\mathbb{Q}(\tau > T \mid \tau > t) = \frac{\bar{B}^A(t,T)}{B(t,T)}$$

where τ is the rv of the time of default. And thus, given the value of risky bonds and risk-free bonds ($B(t,T)$):

$$1 - PD = \mathbb{Q}(\tau > T \mid \tau > t) = e^{-s^A(T-t)}$$

★ **Bootstrapping the spreads.** Bootstrapping is an iterative process that aims at extracting zero-coupon rates from bonds with coupons. Suppose firm A has n bonds $(\bar{B}_1^A, \bar{B}_2^A, ..., \bar{B}_n^A)$ of nominal 1, with a coupon rate c.

- from $\bar{B}_1^A$, we know that:$\bar{B}_1^A = \frac{1+c}{(1+r_1^A)^1}$ and thus that $r_1^A = \frac{1+c}{B_1^A} - 1$;
- from $\bar{B}_2^A$, we know that:$\bar{B}_2^A = \frac{c}{(1+r_1^A)^1} + \frac{1+c}{(1+r_2^A)^2}$, from which, knowing r_1^A, we can extract the value of r_2^A;
- going on like this, we can extract the value of $(r_1^A, r_2^A, ..., r_n^A)$.

Of course this technique can be applied to risky or non-risky bonds.

► **Reduced form models.** Reduced-form models consist in modeling the conditional law of the random time of default:

$$\tau < t + \mathrm{d}t \mid \tau > t$$

The most applied reduced-form models assumes that the probability of default is the product of the infinitesimal time step with a so-called *default intensity* (considered constant here):

$$\mathbb{Q}(\tau < t + \mathrm{d}t \mid \tau > t) = \lambda \times \mathrm{d}t$$

which is equivalent to say that: the random time of default follows an exponential law of rate λ.
Thus, the survival probability of the studied counterparty is:

$$\mathbb{Q}(\tau > t) = \exp(-\lambda t)$$

which is consistent with the no-arbitrage formula introduced earlier.

★ **Hazard rate modeling.**

- **Constant**: Time homogeneous Poisson Process;
- **Deterministic**: Time deterministic inhomogeneous Poisson Process;
- **Stochastic**: Time-varying and stochastic Poisson Process as the Cox, Ingersoll, Ross (CIR) model.

► **Implied probability of default taking into account recovery.** Let R be the recovery rate, the implied probability of default taking into account recovery is:

$$\mathrm{PD} = \frac{1 - \frac{\bar{B}(0,T)}{B(0,T)}}{1 - R}$$

► **The Expected Loss (EL).** The Expected Loss (EL) on a credit exposure can be split in three parts:

- **PD**: the Probability of Default (see above);
- **LGD**: the Loss Given Default is equal to $1-R$, where R is the recovery rate, that is the proportion of the exposure that the lender retrieves;
- **EAD**: the Exposure At Default, that can be fixed (for bullet bonds for example[a]) or not (exposure of derivatives towards a counterparty for example).

Resulting in:

$$\mathrm{EL} = \mathbb{E}(\mathrm{EAD} \times \mathbb{1}_{\{\tau<M\}} \times \mathrm{LGD}) \underbrace{=}_{\text{assump.}} \mathbb{E}(\mathrm{EAD}) \times \underbrace{\mathbb{E}(\mathbb{1}_{\{\tau<M\}})}_{\text{PD}} \times \mathbb{E}(\mathrm{LGD})$$

[a] Bonds which notional is reimbursed at the maturity

PRICING CDS

Lecture 1

The value of the fixed leg of a CDS is:

$$\mathrm{Fixed}(0,T) = s(0,T)\frac{1-e^{-(r+\lambda)T}}{r+\lambda}$$

The value of the floating leg of a CDS is:

$$\mathrm{Floating}(0,T) = (1-R)\frac{\lambda}{\lambda+r}\left(1-e^{-(\lambda+r)T}\right)$$

► **The spread of a CDS.** The spread of a CDS is:

$$s = \lambda(1-R)$$

► **The sensitivity of a CDS.** The sensitivity of the MtM is the risky duration, DV:

$$\mathrm{DV}(t,T,\lambda) = \frac{1-e^{-(r+\lambda)(T-t)}}{r+\lambda}$$

► **Valuation of a CDS.** Let s_0, be the spread in $t=0$, and s_t, the spread today, in t.
The Present Value of the protection seller is:

$$PV(s_0,s_t) = \mathrm{DV}(0,t,\lambda)(s_0-s_t)$$

★ **Other CDS-like products.**

- **CMCDS**: Constant Maturity Credit Default Swaps are like CDS except that the premium paid by the protection buyer is calculated every 3 months (or 6 months) based on the spread of the reference entity at that time.
- **ABSCDS**: Asset-Backed Security Credit Default Swaps are CDS whose reference is not an entity, but an ABS (see below).

TRANSITION MATRICES

Lecture 2

★ **Ratings.** Ratings are an evaluation of the credit risk of a debtor, performed by credit rating agencies.

Moody's	S&P	Fitch	Rating description
Aaa	AAA	AAA	Prime
Aa1	AA+	AA+	High grade
Aa2	AA	AA	
Aa3	AA-	AA-	
A1	A+	A+	Upper medium grade
A2	A	A	
A3	A-	A-	
Baa1	BBB+	BBB+	Lower medium grade
Baa2	BBB	BBB	
Baa3	BBB-	BBB-	
Ba1	BB+	BB+	Non-investment grade / Speculative
Ba2	BB	BB	
Ba3	BB-	BB-	
B1	B+	B+	Highly speculative
B2	B	B	
B3	B-	B-	
Caa1	CCC	CCC	Substantial risks
Caa2	CCC	CCC	
Caa3	CCC	CCC-	
Ca	CC	CC	Extremely speculative
C	C	C	Default imminent
C	RD	DDD	In default

► **Transition matrices.** In credit risk, a transition matrix, $\boldsymbol{M}_{t,t+1} = (m_{ij})_{ij}$, is a matrix where:

$$m_{ij} = \mathbb{P}(\mathrm{Grade}_{t+1} = j \mid \mathrm{Grade}_t = i)$$

► **The generator of an homogeneous Markov chain.** The generator for a Markov chain $(\boldsymbol{M}_{t,t+n})_n$ is the matrix $\boldsymbol{Q}$ so that:

$$\forall (t,T), \quad \boldsymbol{M}_{t,T} = \exp((T-t)\boldsymbol{Q}) \quad \text{with } \exp(\boldsymbol{A}) = \sum_{n\geq 0}\frac{\boldsymbol{A}^n}{n!}$$

Would such a matrix exist, we have:

$$\boldsymbol{Q} = \sum_{n>0}(-1)^{n-1}\frac{(\boldsymbol{M}_{t,t+1}-\boldsymbol{I})^n}{n}$$

▲ **Markovian property.** The existence of such a generator matrix is based on the assumption of the Markov propriety that states (in the discrete case) that:

$$\mathbb{P}(\mathrm{Grade}_{t+1} = j \mid \mathrm{Grade}_t = i) = \mathbb{P}(\mathrm{Grade}_{t+1} = j \mid \mathrm{Grade}_t = i, \mathrm{Grade}_{t-1} = i_{t-1}, \ldots, \mathrm{Grade}_{t-h} = i_{t-h})$$

which is not observed in practice.

STATISTICAL TOOLS TO ASSESS CREDIT RISK

Lecture 2

► **Logistic regression.** The logistic regression model can be defined the following ways:

$$p(X) = \mathbb{P}(Y=1|X) = \frac{e^{\beta_0+\beta_1X_1+\ldots+\beta_pX_p}}{1+e^{\beta_0+\beta_1X_1+\ldots+\beta_pX_p}}$$

or equivalently

$$\ln\left(\frac{p(X)}{1-p(X)}\right) = \beta_0+\beta_1X_1+\ldots+\beta_pX_p$$

where $X = \left(X_1,\ldots,X_p\right)$

★ **Pros and Cons of the logistic regression.**

- **Pros**: no close-formula but easy to fit with MLE, can easily be interpreted thanks to odds ratios.
- **Cons**: cannot capture non-linear relationships between the variables.

★ **Classification trees.** A classification tree is a model with a tree-like structure. It contains nodes and edges/branches. It is grown through a recursive binary splitting procedure:

- Step 1: We select the predictor X_k and the cutting point s in order to split the predictor space into the two regions $\{X, X_k < s\}$ and $\{X, X_k \geq s\}$ that gives the greatest decrease in the criterion we want to minimize (Gini index for example).
- Step 2: We repeat step 1 but instead of splitting the whole predictor space we split one of the two regions identified at step 1. The predictor space is now divided into three regions.
- Step 3: The regions are then recursively split until no split can decrease the criterion.

★ **Pros and Cons of decision trees.**

- **Pros**: Can be easily interpreted and visualised, allows for non-linear predictions.
- **Cons**: Exhibit high variance, can be biased when the sample is unbalanced.

▲ **Bagging and correlation.** Bagging consists in training several trees on bootstrapped samples from the original training set. The trees are then aggregated by choosing the most common prediction among the trees' predictions or any suitable voting rule.

- The resulting classifier exhibits less variance than a single trees
- The gain in variance is limited since the trees are correlated

► **Random Forests.** A Random Forest algorithm is a bagging procedure with trees trained as follows:

- A random sample is drawned from the training set
- At each knot the split is made using only a random subset of the predictors

Using only a subset of the predictors yields less correlated trees than a standard bagging algorithm.

★ **Pros and Cons of Random Forest.**

- **Pros**: Exhibit less variance than basic trees or bagging and are less prone to overfitting.
- **Cons**: Have no straightforward interpretation.

► **Support Vector Machine.** SVM is a binary classifier that provides a decision boundary in the predictor space which is as far as possible from the two types of observations to separate. Using the so-called "kernel trick", it can provide highly non-linear decision boundaries.

★ **Pros and Cons of Support Vector Machine.**

- **Pros**: Can capture highly non-linear patterns and can be very robust to small change in data.
- **Cons**: Is not easy to interpret, does not provide a straightforward PD, and is prone to overfitting.

STRUCTURAL MODELS

Lecture 3

► **Merton model.** The Merton model is a structural model that assumes that the default occurs when the value of the firm (V_t) is inferior to D the amount of debt of the firm, at maturity T of the debt. In this model, the value of the firm follows the following process:

$$\frac{dV_t}{V_t} = r\,dt + \sigma\,dW_t$$

Using option value theory after noticing being a shareholder is like having a European call option of strike D, maturity T, on V_t, one can find the value of the equity, of the debt and thus, the spread of the firm and its probability of default.

► **Leland model.** The Leland model is a structural model that assumes that default occurs when the value of the assets of the firm goes under a certain threshold K. The assets follows the following process:

$$\frac{dA_t}{A_t} = (r-\delta)dt + \sigma dW_t$$

where δ is the dividend rate.

Using Laplace transform, one can deduce the value of the debt, the equity and the firm, today. Moreover, one can find the optimal amount of capital in the balance sheet and the optimal coupon to pay in order to optimize the value of the equity of the firm.

▲ **Structural models limits.** Structural models carry interesting economic interpretations, but for single-name models they are two simplistic to be used for pricing or risk measurement. Statistical models are mostly used in the historical probability world and reduced-form models in the risk neutral world.

PORTFOLIO MODELS

Lecture 4

► **Vasicek model.** The Vasicek model gives the loss distribution of a portfolio of defaultable assets.
Let us suppose we have a countable infinite number of bonds (loans, mortgages, etc.) of equal nominal, same maturity, same probability of default at maturity (PD), and a same recovery rate (R). We assume that bond i defaults when the latent variable $R_i < s$, where s is a common latent threshold for all bonds. Moreover, we assume that:

$$\forall i \in \mathbb{N}, \qquad R_i = \underbrace{\sqrt{\rho}}_{\substack{\text{corelation}\\\text{factor}}} \underbrace{F}_{\substack{\text{systemic}\\\text{factor}}} + \sqrt{1-\rho} \underbrace{e_i}_{\substack{\text{idiosyncratic}\\\text{factor}}}$$

with $(e_i)_{i\in\mathbb{N}}$ and F are standard normal variables, and thus $(R_i)_{i\in\mathbb{N}}$ are standard normal and correlated.
We know that $\text{PD} = \mathbb{Q}(R_i < s) = \underbrace{\Phi}_{\substack{\text{Normal}\\\text{cdf}}}(s)$ and thus:

$$s = \Phi^{-1}(PD)$$

We thus have that for the random variable of the losses of the portfolio expressed as a percentage:

$$\begin{aligned} L \mid F &= \lim_{N\to+\infty} \frac{1-R}{N} \sum_{i=1}^{N} \mathbb{1}_{\{R_i < s\}} \\ &= \lim_{N\to+\infty} \frac{1-R}{N} \sum_{i=1}^{N} \mathbb{1}_{\left\{e_i < \frac{\Phi^{-1}(PD)-\sqrt{\rho}F}{\sqrt{1-\rho}}\right\}} \\ &\underbrace{=}_{\substack{\text{Law of}\\\text{large numbers}}} (1-R)\Phi\left(\frac{\Phi^{-1}(PD)-\sqrt{\rho}F}{\sqrt{1-\rho}}\right) \end{aligned}$$

Note that L is conditioned by the value of F, the stochastic systemic factor.

► **Copulas.** A copula C, is a function ($[0;1]^d \to [0;1]$) that allows to model dependencies:

$$\forall (x_1,\dots,x_d) \in \mathbb{R}^d, \quad F(x_1,\dots,x_d) = C(F_1(x_1),\dots,F_d(x_d))$$

Sklar's theorem asserts that from any continuous multivariate distribution G, a copula can be deduced with the following formula:

$$\forall (u_1,\dots,u_d) \in [0;1]^d, \quad C(u_1,\dots,u_d) = F(F_1^{-1}(u_1),\dots,F_d^{-1}(u_d))$$

where $F_1,\dots,F_d$ are the marginal distributions of the $d-$dimensional distribution F.

★ **The Gaussian copula.**. In the Gaussian case, we have:

$$\forall (u_1, ..., u_d) \in [0;1]^d, \quad C^{\mathcal{N}}_{\mathbf{R}}(u_1, ..., u_d) = \Phi_{\mathbf{R}}(\Phi^{-1}(u_1), ..., \Phi^{-1}(u_d))$$

▲ **Why and how to use copula?**

- **Why?** Thanks to copula, one can design complex dependencies between the defaults (tail dependencies, asymmetric dependencies, etc.).
- **How?** Most copula usage requires numerical simulations: (i) the copula is simulated to get uniform observations whose structure of dependence is the one of the copula, (ii) by composing each uniform simulation by the considered univariate distribution, one gets the required marginal distribution with the structure of dependence of the copula.

The copulas most used are Gaussian, Student, Archimedean, Clayton, Franck, Gumbel copulas.

FTD, CLN, ABS, CDO, CSO

Lecture 4

► **First To Default derivative (FTD).** A FTD is a *multiname* Credit Default Swap: the reference is a *reference pool* (usually between 5 and 10 reference entities), and the first credit event among the reference entities triggers the FTD.

► **Asset Backed Securities (ABS).** An ABS is a security whose income payments and hence value are derived from and collateralized (or "backed") by a specified pool of underlying assets placed in a Special Purpose Vehicle (SPV), that is a legal entity created specifically for the purpose of holding the assets. For instance, RMBS (Residential Mortgage Backed Security) are ABS for which the underlying assets are mortgages.

► **Spread of the tranche of an ABS.** To assess the spread of a CDO, that is a tranche of an ABS, let us consider the case of a given tranche whose attachment point is A and detachment point D. We denote L_t the distribution loss of the underlying portfolio in t, and $L_t^{[A;D]}$ the distribution loss on the considered tranche.
The cash-flows for the buyer of the tranche are fixed and equal to:

$$JF^{[A;D]}(0,T) = s^{[A;D]} \underbrace{\int_0^T e^{-rt}(D - A - EL_t^{[A;D]})dt}_{DV^{[A;D]}(0,T)}$$

where $EL_t^{[A;D]} = E(L_t^{[A;D]})$
The cash-flows for the seller of the tranche are variable and equal to:

$$JV^{[A;D]}(0,T) = EL_T^{[A;D]} e^{-rT} + r\int_0^T e^{-rt} EL_t^{[A;D]} dt$$

And thus, the spread of the tranche is:

$$s^{[A;D]} = \frac{JF^{[A;D]}(0,T)}{DV^{[A;D]}(0,T)}$$

NB: for the loss distribution, one can use any portfolio model, Vasicek's being the one used most often.

▲ **Implicit correlation – Base correlation.** Let us consider the tranche of an ABS, with attachment point A, detachment point D and spread $s^{[A;D]}$.

- **Implicit correlation**: the implicit correlation of the considered tranche, is the correlation $\rho^{[A;D]}$ to put into the pricing model to find $s^{[A;D]}$;
- **Base correlation**: the base correlation of A (and of D) is the implicit correlation $\rho^{[0;A]}$ (and $\rho^{[0;D]}$) to put into the pricing model to find the spreads of the – theoretical – equity tranches $s^{[0;A]}$ (and $s^{[0;D]}$).

One can show that:

$$s^{[A;D]} = \frac{JV^{[0;D]}(\rho^{[0;D]}) - JV^{[0;A]}(\rho^{[0;A]})}{DV^{[0;D]}(\rho^{[0;D]}) - DV^{[0;A]}(\rho^{[0;A]})}$$

ensuring a bijective relationship between base correlations and spreads of CDO tranches.

▲ **Different kinds of credit enhancement.** There are several way to improve the credit profile of an ABS / a tranche of an ABS (CDO):

- **Excess spread**: the received rate is higher than the served one;
- **Overcollateralization**: the face value of the underlying loan portfolio is larger than the security it backs;
- **Monolines and wrapped securities**: CDS on the underlying assets are bought to monolines in order to cover the losses.

► **Collateralized Synthetic Obligation (CSO).** A form of CDO that does not hold assets like bonds or loans but invests in CDS or other non-cash assets to gain exposure to a portfolio of fixed income assets.

★ **Trading and hedging correlation.** CDOs or CSOs are sensitive to the correlation of default of the underlying bonds or loans. Thus, an investor can either be:

- **Long correlation**: when he buys the equity tranche;
- **Short correlation**: when he buys the super senior tranches.

In fact, if the correlation increase, the probability of default of none (or all) of the underlying assets increases reducing the risk of loss for the equity tranche buyers (and increases the risk of losses of the super senior tranches buyers).

► **Collateralized Debt Obligation Square (CDO2).** A CDO square is an ABS whose assets are CDO, mostly mezzanine tranches of several RMBS.

REGULATORY APPROACH TO CREDIT RISK

Lecture 5

► **Credit Risk Weighted Assets (RWA).** Risk Weighted Assets are computed by weighting assets following a regulatory formula $f_{\text{Credit RWA}}(\text{PD}, \text{LGD}, \text{EAD})$.
Adding Credit RWA, to Market RWA and Operational RWA, we get the RWA of the banks, and can thus compute the capital ratio:

$$\text{capital ratio} = \frac{\text{capital}}{\text{RWA}}$$

▲ **Different approaches**. There are three approaches to compute credit RWA:

- **The Standard Approach**: the RWA is the product of a given Risk Weigh multiplied by EAD;
- **IRBF (Internal Rating Based Foundation)**: the bank can estimate PD; LGD and EAD depends on regulatory formulas that depend on the considered asset;
- **IRBA (Internal Rating Based Approach)**: PD, LGD and EAD are computed by internal models of the bank.

RETURN AND VALUE CREATION FOR A BANK

Lecture 5

► **Return On Equity (ROE).** The Return On Equity of a Business Line is:

$$\text{ROE} = \frac{\text{Net Income of the BL}}{\text{Regulatory Capital allocated to the BL}}$$

► **Risk Adjusted Return of Capital (RAROC).** The Risk Adjusted Return On Capital of a Business Line is:

$$\text{RAROC} = \frac{\text{Net Income of the BL} - \text{Average loss of the BL}}{\text{Economic Capital allocated to the BL}}$$

► **Weight Average Cost of Capital (WACC).** The Weighted Average Cost of Capital is:

$$\text{WACC} = (r + k_1)T_1 + (r + k_2)T_2 + (r + k_d)D$$

where, T_1/r_1, T_2/r_2 and D/r_D is the proportion / the cost (spread) of Tier 1 capital, Tier 2 Capital and debt in the liabilities of the bank and r the risk-free rate.

▲ **WACC and hurdle rate..** The WACC is the hurdle rate of bank activities, that is, the minimum return necessary to create value.

► **Economic Value Added (EVA).** The Economic Value Added for a bank is:

$$\begin{aligned} \text{EVA} &= \text{Net Income of the bank} \\ &- \text{Average loss of the bank} \\ &- \text{WACC} * \text{Liabilities} \end{aligned}$$

Another definition of the Economic Value Added for a bank is:

$$\begin{aligned} \text{EVA} &= \text{Net Income of the bank} \\ &- \text{Average loss of the bank} \\ &- k * \text{Economic Capital} \end{aligned}$$

where k is the cost of capital, that is: $k = (r + k_1)T_1 + (r + k_2)T_2$.

► **Risk Adjusted Returns On Risk Adjusted Capital (RARORAC).** The Risk Adjusted Return On Risk Adjusted Capital for a bank is:

$$\text{RARORAC} = \text{RAROC} - k$$

Another definition of the RARORAC is:

$$\text{RARORAC} = \text{RAROC} - k \times \frac{\text{Allocated Economic Capital}}{\text{Used Economic Capital}}$$

► **Cost of capital – A CAPM approach.** The Capital Asset Pricing Model states that the average return of the stock i, $\mathbb{E}(r_i)$, follows:

$$\mathbb{E}(r_i) = \beta_i(\mathbb{E}(r_M) - r_f) + r_f$$

where $\beta_i = \rho_{i,M}\frac{\sigma_i}{\sigma_M}$, r_M is the return of the whole stock market, σ_M is the volatility of the whole stock market, σ_i is the volatility of the stock i, $\rho_{i,M}$ is the correlation between the returns of the market and the ones of i, r_f is the risk-free rate.

► **Cost of capital – The Gordon-Shapiro Approach.** The Gordon-Shapiro approach states that the valuation of a firm, P, is a function of the expected growth g of its cash-flows (D_t) and the expected return k of the shareholders:

$$P = \sum_{t=1}^{\infty} \frac{D_t}{(1+k)^t} = \frac{D_1}{k-g}$$

with $D_t = D_{t-1} \times (1+g)$

★ **What is the cost of equity?.** Shares, contrary to bounds, do not specify the future returns. Nonetheless, their expected return, necessary to compute the WACC, can be *extracted* from the market using the CAPM approach ($\mathbb{E}(r_i)$) or the Gordon Shapiro approach (k).

COUNTERPARTY RISK

Lecture 6

► **Counterparty risk – Definition.** The counterparty risk is defined as the risk that the counterparty to a transaction defaults before the final settlement of the transaction's cash-flows. It can be a bilateral risk as the exposure may vary with the market conditions.

▲ **What drive the counterparty risk?.** Counterparty risk is driven by (i) OTC contract's market value risk drivers, (ii) the counterparty credit spread, and (iii) the correlation between the underlying and the probability of default of the counterparty (Wrong Way Risk – WWR – and Right Way Risk – RWR).

★ **Use of counterparty risks measurement.**

- **Pricing**: to take into this risk when pricing a derivative (Counterparty Value Adjustment – CVA);
- **Risk management**: for internal and regulatory purposes (SIMM, KCVA, PFE, etc. – see below).

► **Exposures – EE, PFE, MPFE, EPE, EEPE.** $V(t)$ denotes the market value of a derivative, at time t. Counterparty exposure is equal to $E(t) = V(t)^+ = \max(0, V(t))$.

Expected Exposure (EE).

$$\text{EE}(t) = \mathbb{E}^{\mathbb{Q}}(E(t))$$

Potential Future Exposure (PFE).

$$\text{PFE}_\alpha(t) = q_\alpha(\text{E}(h))$$

Maximum Potential Future Exposure (MPFE).

$$\text{MPFE}(t) = \max_{h<s}(\text{E}(h))$$

Effective Positive Exposure (EPE).

$$\text{EPE}(t) = \frac{1}{t}\int_0^t \text{EE}(s)\text{d}s$$

Expected Effective Positive Exposure (EEPE).

$$\text{EEPE}(t) = \frac{1}{t}\int_0^t \max_{h<s}(\text{EE}(h))\text{d}s$$

PFE is then considered as the EAD in the regulatory formula used to compute RWA.

► **CVA – Credit Value Adjustment.**

$$\text{CVA} = (1-R)\int_0^T \mathbb{E}^{\mathbb{Q}}(E(s))\text{d}S_C(s)$$

Where S_C is the survival function of the counterparty.

★ **Two ways to mitigate counterparty risk:**

- **Netting**: in presence of multiple trades with a counterparty, netting agreements allow, in the event of default of one of the counterparties, to aggregate the transactions before settling claims ($E(t) = (\Sigma V(i))^+ \neq \Sigma(V(i))^+$).
- **Collateralizing**: collateral is a property or other assets that a counterparty offers as a way for the counterparty to secure the

exposure.

▲ **Other counterparty risk metrics**. There are several other counterparty risk metrics such as:

- **SIMM**: SIMM stands for Standard Initial Margin Model and is used to compute the initial margin of non-cleared derivatives;
- **KCVA** is a VaR estimate of the CVA required by Basel III that impose a capital charge.

★ **IRC and CRM, credit market risk metrics:** IRC and CRM are credit market risk metrics that are not counterparty risk metrics as they measure the credit risk due to the potential default(s) of the underlying reference(s) on derivative products. With the VaR and the stressed VaR, these measure are part of the market RWA.

► **IRC – Incremental Risk Charge** The IRC is a risk metrics that captures risk due to adverse rating migrations on vanilla credit securities such as bonds and CDS on corporates and sovereigns within the trading book.

► **CRM – Comprehensive Risk Measure**

- The CRM is a risk metrics that, as the IRC, captures the risks due to adverse rating migrations on credit securities. It applies to credit correlation portfolios (CDO, CLO, CBO, etc.) within the trading book.
- The CRM also captures risks due to credit spread, recovery rates and base correlations variations.