Final Test – 2018-2019

Credit Risk

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Exercise 1: Counterparty Risk.

1. The table below gives the current mark-to-market values of 6 OTC contracts between the banks X and Y:

	P_1	P_2	P_3	P_4	P_5	P_6
X	5	6	-3	0	-12	13
Y	-5	-7	3	0	10	-15

The table above must be read as follows: bank X declared a mark-to-market value of 5 for contract P_1, whereas bank Y declared a mark-to-market equal to -5.

a. Give a reason for the inconsistencies in the table between the reported exposures of banks X and Y.

> Bank X and Y might use different pricing models and therefore have different valuation.

b. Compute the exposures at default for bank X and bank Y.

$$\text{EAD} = \sum_{k=1}^{6} \max(P_k, 0) \tag{1}$$

$$\begin{aligned}\text{EAD}_A &= 5 + 6 + 13 = 24\\ \text{EAD}_B &= 3 + 10 = 13\end{aligned}$$

c. Compute the same exposures at default if a netting agreement exists between bank X and Y for the 6 contracts.

> In case of a netting agreement on the 6 contracts, we have:

$$\text{EAD} = \max\left(\sum_{k=1}^{6} P_k, 0\right) \tag{2}$$

> and therefore:

$$\begin{aligned}\text{EAD}_A &= \max(5 + 6 - 3 + 0 - 12 + 13, 0) = 9\\ \text{EAD}_B &= \max(-5 - 7 + 3 + 0 + 10 - 15, 0) = 0\end{aligned}$$

d. Compute the same exposures at default if two netting agreements exist between bank X and Y for the contracts P_1 and P_2 on one side, and for P_3, P_4 and P_6, on the other side.

If N_i denote one of the M disjoint netting agreements, we have:

$$\mathrm{EAD} = \sum_{i=1}^{M} \max\left(\sum_{k\in N_i} P_k, 0\right) \tag{3}$$

and therefore:

$$\begin{aligned}
\mathrm{EAD}_A &= \max(5+6,0)+\max(-3-12+13,0)+\max(-12,0) = 21\\
\mathrm{EAD}_B &= \max(-5-7,0)+\max(3+10-15,0)+\max(10,0) = 10
\end{aligned}$$

2. Let $e(t)$ denote the exposure at default of an OTC contract of maturity T. We assume that the current date is $t=0$. We suppose that the exposure at default at each date is defined as follows:

$$e(t) = N\sigma\sqrt{t}U \tag{4}$$

with $0 \le U \le 1, \mathbb{P}(U \le u) = u^{\gamma}$ and $\gamma > 0$.

a. What is the maximum exposure at default at time t? until maturity?

The maximum value can be reached at T with $U=1$ that is to say $N\sigma\sqrt{T}$. At time t, the maximum possible exposure would be $N\sigma\sqrt{t}$.

b. Compute the Potential Future Exposure $\mathrm{PFE}_\alpha(0;t)$.

Let's first compute the cummulative distribution function of $e(t)$:

$$\begin{aligned}
\forall x \in \left[0, N\sigma\sqrt{t}\right], \qquad F_t(x) &= \mathbb{P}(e(t) \le x)\\
&= \mathbb{P}\left(U \le \frac{x}{N\sigma\sqrt{t}}\right)\\
&= \left(\frac{x}{N\sigma\sqrt{t}}\right)^{\gamma}
\end{aligned}$$

We then deduce that:

$$\mathrm{PFE}_\alpha(0;t) = F_t^{-1}(\alpha) = N\sigma\sqrt{t}\alpha^{\frac{1}{\gamma}} \tag{5}$$

c. Compute the Expected Exposure $\mathrm{EE}(0;t)$.

$$\mathrm{EE}(0;t) = \mathbb{E}[e(t)] = \int_0^{N\sigma\sqrt{t}} x\,\mathrm{d}F_t^{-1}(x) = \frac{\gamma}{\gamma+1}N\sigma\sqrt{t} \tag{6}$$

d. Compute the Expected Effective Positive Exposure $\mathrm{EEPE}(0;h)$.

$$\begin{aligned}\text{EEPE}(0;h) &= \frac{1}{h}\int_0^h \max_{u\in[0,t]}(\text{EE}(0;u))\,\mathrm{d}t \\ &= \frac{1}{h}\int_0^h \frac{\gamma}{\gamma+1}N\sigma\sqrt{t}\,\mathrm{d}t \\ &= \frac{\gamma}{\gamma+1}N\sigma\left[\frac{2}{3}t^{3/2}\right]_0^h \\ &= \frac{2\gamma}{3(\gamma+1)}N\sigma\sqrt{h}\end{aligned}$$

e. Deduce from the previous question a proxy of the regulatory exposure at default.

> The regulatory exposure at default is equal to $1.4\times \text{EEPE}(0;1)$

Exercise 2: Non-linearity of ABS tranches and the Vasicek model.

The purpose of the exercise is to study the limits of the Vasicek model when it comes to assess non-linear products such as tranches of ABS (Asset Backed Securities).

1. A tranche of ABS is said to be non-linear.

a. Can you recall what an Asset Backed Security (ABS) is?

> An asset-backed security (ABS) is a financial security collateralized by a pool of assets such as loans, leases, credit card debt, royalties or receivables. For investors, asset-backed securities are an alternative to investing in corporate debt.

b. Can you recall the pay-off of the tranche of an ABS with attachment point A and detachment point B? Plot the pay-off in function of the percentage of losses of the portfolio at maturity.

> The owner of a tranche of ABS earns each year the amount of interests that are fixed at the origination of the contract but losses the notional if the percentage of notional in defaults is above the attachment point. Would it be over the detachment point, the owner would loss all the notional and only get the interests up to the year the defaults went over the detachment point.

c. Deduce from question 1.b. why the tranche of an ABS is said to be a non-linear product?

> If we draw the pay-off in function of the percentage of default in the portfolio for a tranche with attachment point A and detachment D, we can see that it is not a straight line. The pay-off is thus non-linear.

2. The Vasicek model gives the loss distribution of a portfolio of defaultable assets.
Let us suppose we have a countable infinite number of bonds (loans, mortgages, etc.) of equal nominal, same maturity, same probability of default at maturity (PD), and a same recovery rate (R). We assume that bond i defaults when the latent variable $R_i < s$, where s is a common latent threshold for all bonds. Moreover, we assume that:

$$\forall i \in \mathbb{N}, \qquad R_i = \underbrace{\sqrt{\rho}}_{\substack{\text{corelation}\\\text{factor}}}\underbrace{F}_{\substack{\text{systemic}\\\text{factor}}} + \sqrt{1-\rho}\underbrace{e_i}_{\substack{\text{idiosyncratic}\\\text{factor}}}$$

with $(e_i)_{i\in\mathbb{N}}$ and F are standard normal variables, and thus $(R_i)_{i\in\mathbb{N}}$ are standard normal and correlated.
We know that PD $= \mathbb{Q}(R_i < s) = \underbrace{\Phi}_{\substack{\text{Normal}\\ \text{cdf}}}(s)$ and thus:

$$s = \Phi^{-1}(PD)$$

We thus have that for the random variable of the losses of the portfolio expressed as a percentage:

$$\begin{aligned} L \mid F &= \lim_{N\to+\infty} \frac{1-R}{N}\sum_{i=1}^{N} \mathbb{1}_{\{R_i<s\}} \\ &= \lim_{N\to+\infty} \frac{1-R}{N}\sum_{i=1}^{N} \mathbb{1}_{\{e_i<\frac{\Phi^{-1}(PD)-\sqrt{\rho}F}{\sqrt{1-\rho}}\}} \\ &\underbrace{=}_{\substack{\text{Law of}\\ \text{large numbers}}} (1-R)\Phi\left(\frac{\Phi^{-1}(PD)-\sqrt{\rho}F}{\sqrt{1-\rho}}\right) \end{aligned}$$

Note that L is conditioned by the value of F, the stochastic systemic factor.

The Vasicek model is based on an homogeneity assumption. Thus, a financial engineer to price an ABS tranche sets parameters in the model with their average value (e.g. average probability of default, average recovery rate).

a. Explain why, because of the homogeneity assumption, one can say that the Vasicek model does not precisely account for "second order effects" (variance of the parameters).

> As probability of defaults and recovery rates are averaged and stay constant in the model, their variability in practice is not retained in the modeling. As the variance of parameter is set to 0, the model does not account for the variance of the parameters. That is why, one can say that the Vasicek model does not precisely account for "second order effects".

b. What consequence does it have on the pricing of ABS tranches?

> Having no variance for the probability of default and recovery rate parameters means that the equity tranche of a portfolio (for instance, let us say the 0 to 3 percents tranche) in which all the PD are for real all equal to the same value (let us say one percent) has the same value as the one of a portfolio in which the distribution of the PD is not a Dirac one, on in which for example 3% of the PD are equal to ten percent, 94% are equal to 1 percent and 3 percents to 0.1%. In both case, the average PD on the whole portfolio is the same, but for sure, the equity tranche of the second one is worth less as the probability of default is higher on the equity tranche. Because of the homogeneity hypothesis, this aspect is not taken into account in the Vasicek model. Intuitively, this effect is close to the vega in option pricing.

3. The Vasicek model is based on the assumption that there is an infinity of debtors in the portfolio.

a. Imagine you own a tranche of ABS which thickness is of 5% and that the portfolio is made of 10 assets, each one representing 10% of the total notional. What will happen in case of defaults? What link can you make between this case and the payoff of a digital option (which payoff is either one or zero depending on the underlying asset's value at maturity)?

> In that case, the payoff is either the cumulative interests rates or zero, as a default would consume the whole tranche from attachment point A to D. It is a binary pay off like for a digital option.

b. Intuitively, compare the sensitivy of the price of this tranche towards the latent variable of the ith debtor R_i in that case, and in the case where there is an infinity of debtors.

> In that case, would R_i grows higher, the probability that all the notional of the tranche is consumed instantly is superior, than in the other case, in which, the risk is more diversified and the effect would not be sudden as it would need many more defaults.

c. Conclude about the limit of the infinity assumption.

> The bigger the chunks in the portfolio, the less diversification applies and the more the price is sensitive to the variation of the latent variables of each debtor. This effect is similar to the gamma in option pricing (second derivative sensitivity on the underlying asset). A large component of binary option is due to this effect, due to the jump in the pay-off of the security.

4. The Vasicek model cannot take into account tail dependence.

a. What does it mean in an economic perspective?

> On a mathematical perspective, it means that the correlation between two latent variables stays the same, no matter the value they have.
> Suppose now we have two pools of debtors, pool A and B. On an economic perspective, it means that the probability of default in pool B is not amplified by the fact that the probability of default is increasing in pool A. Given the interconnectedness of agents in the economy, one might want that the model accounts for an increase of the dependence between defaults, when many defaults already happened, to take into account what might happen in case of an economic turmoil.

b. What consequence can you spot in the pricing of ABS tranches?

> Because of the absence of tail dependence, extreme scenarios are less likely. Thus, the price of the last tranche (higher quality tranche) does not take into account these events: they might be underpriced.

c. Can you remind what base and implied correlations are? Can you recall the phenomenon that is observed on base correlations?

> The base correlation in attachment point A is the theoretical correlation to apply in the Vasicek model to get the price of the tranche between 0 and A. The implied correlation between A and detachment point D, is the correlation to use in the Vasicek model to get the price quoted on the market of the tranche A to D.
> Even if correlations are supposed to be constant for all tranches in the Vasicek model, the implied and base correlation, for one ABS, changes across tranches.

d. Can you link the tail dependence assumption and the phenomenon observed on the market on base correlations?

> As the market wants to account for extreme events in the pricing of certain tranches, the correlation for each tranche is different to account in a different manner for extreme events (increasing the correlation for the last tranche for example).

Exercise 3: Credit risk and the real-world versus risk-neutral probability dilemma.

The purpose of this exercise is to link (i) credit market structure, (ii) with the choice of probability retained for models, (iii) with the risk mitigation techniques used by banks (hedging or diversifying), (iv) with the accounting rules and (v) with the regulatory requirements. You should be able to answer each question with one or two sentences.

1. Most financial market models are based on risk-neutral probabilities. For credit models, a large part of them are based on real-world probabilities.

a. Can you recall the difference between risk-neutral and real-world probabilities?

> Real-world probabilities come from statistical observations of passed event. On the contrary, risk-neutral probabilities originate from market prices. Indeed, market prices provide a forward costs to hedge a risk, to be risk neutral to it. Risk-neutral probabilities are weights of future events dependent on the costs to hedge them.

b. If there is no public market, there is no risk-neutral probability. Comment on that.

> Given the previous question, one needs a market to imply risk neutral probabilities. Without market, there is no way to estimate risk neutral probabilities.

c. Can you recall which credit activities within the bank use risk-neutral probabilities? Which ones use real-world probabilities?

> Most regular financing activities (e.g. project finance, debt finance, consumer loans, mortgages) use real world probabilities (there is no alternative as there is no market). Market activities implying credit or counterparty risk use risk neutral probabilities.

2. To mitigate credit risk, a bank can either hedge the risks or diversify them. Can you map these mitigation techniques with either the risk-neutral or the real-world probabilities?

> When there is a market exchanging credit risk (through bonds, CDS or others), not can a bank only price and model its risk through risk neutral probability but it can also buy these assets to hedge its risk (by buying assets on the market that will make it insensitive to any change of market parameters). On the other hand, if there is no market, not only must the bank assess its risk based on passed data, but the bank has no other way to hedge its risks than diversification.

3. In class, we studied provisions.

a. Can you recall what a provision is?

> Provisions are balance sheet items representing funds set aside by a company as assets to pay for anticipated future losses.

b. Can you recall how IFRS9 (for credit lending) and IFRS13 (for CVA) require banks to use risk-neutral or real-world probabilities in their provisioning models?

> IFRS 9 requires the bank to use real-world probabilities and IFRS13 risk neutral ones.

4. We also studied KCVA and Credit RWA, both are capital requirements. For the recall, KCVA is based on a Value at Risk of the CVA PL, and Credit RWA on a portfolio model.

a. Can you recall what KCVA is?

> KCVA is a Value-at-risk calculated capital requirement from Basel III designed by the regulator so that banks can face an important move of their IRFS13 CVA risk neutral provisions.

b. Can you recall what Credit RWA are?

> Credit Risk Weighted Assets are a transformed version of the assets of a bank used to calculate the a amount of capital required by the regulator so that the bank can face unexpected losses due to defaults.

c. What is a Value at Risk?

> The Value-at-risk is a percentile on a loss distribution.

d. Why does it make sense that KCVA is based on an PL VaR and Credit RWA on a portfolio model?

> KCVA has been implemented so that unhedged or partially hedged CVA provisions require banks to have more capital. Credit RWA is a portfolio model that accounts for the diversification of loans granted by the bank and is calibrated with real-world parameters. KCVA is used to measure unhedged market exposure. Credit RWA is used to measure the exposure of the bank on credit risk taking into account the types of loans and the effect of diversification (portfolio models). Thus, both KCVA and Credit RWA are coherent with risk mitigating technique used by the bank in the way they assess capital requirements.