Final Test – 2021-2022

Credit Risk

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Master II

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Exercise 1: LCDS.

A LCDS or Loan CDS is a Credit Default Swap which reference is not an obligation but a loan. In almost all the ways, it is equivalent to a CDS except that as loans can be refinanced or reimbursed earlier than their maturity, LCDS can be canceled. The purpose of this exercise is to understand how to price LCDS.

1. Recall the relationship between the premium or spread of a *regular* CDS, s, the recovery rate of the underlying obligation, R and the default intensity of the obligor λ.

2. We saw than a Loan CDS can be canceled if the underlying loan is refinanced or reimbursed by the counterparty. Let us introduce τ_C the random time of the cancellation of the LCDS and τ_D the random time of default of the loanee.Let us define τ_{TM} the random time of the termination of the LCDS (either because of a default of the loanee or a refinancing / reimbursement of the loan) and τ_{TR} the random time of the triggering of the LCDS, that is when the LCDS is activated because of a default of the counterparty before the maturity of the loan or any refinancing / reimbursement and ∞ if the loan is refinanced before maturity.
a. Write the mathematical relationship between both τ_{TM} and τ_{TR} and the time of cancellation (τ_C) and default (τ_D).
b. Let denote $S_C(t)$ and $S_D(t)$ the survival functions of the cancellation time and default time of a LCDS (that is $S_C(t) = \mathbb{P}(\tau_C > t)$ for instance). Show that the cumulative distribution F_{TM} and F_{TR}, respectively of the termination time and triggering time are equal to: $F_{TM}(t) = \mathbb{P}(\tau_D < t, \tau_C < t)$ and $F_{TR}(t) = \mathbb{E}(\mathbb{1}_{\{\tau_D<t\}} \times S_C(\tau_D))$.

3. We call s the spread of the LCDS, M the maturity of the loan, R the recovery rate on the loan, and r the constant discount rate. We assume that the nominal of the reference loan is 1. Show that the value of the Fixed Leg and the Variable Leg (FL_t and VL_t) are equal to:

$$FL_t = \int_t^M s e^{-r(h-t)} dS_{TM}(h)$$

$$VL_t = \int_t^M (1-R) e^{-r(h-t)} dS_{TR}(h)$$

4. In this question, we assume that the time of cancellation and time of default respectively follow an exponential law of parameter c and λ and are independent. Show that the spread of the LCDS is the same as a regular CDS (same expression as in question 1).

5. We drop the independence assumption between the time of cancellation and the time of default.
a. Discuss the validity of the independence assumption made in question 4.
b. How would you model dependence between the cancellation time and the default time?

Exercise 2: The climate component of credit risk.

In this exercise, we will focus on the Merton model that we will adapt so as to add a climate risk component. Both questions 4 and 5 focus on transition risk impact while question 6, 7 and 8 will allow us to propose a naive model for physical risk impact on credit risk.

Notations are as follows:

- V_t is the assets' market value at time t, V_t dynamic is:

$$\frac{dV_t}{V_t} = rdt + \sigma dW_t,$$

- The firm is financed by Debt and equity,
- Debt principal is equal to D and is to be repayed at time T.

1. Recall the main assumptions of the Merton model.

2. [Bonus points] We note D the amount of debt in the balance sheet in t, D_t its value, and E_t the value of equity (in t). Show that:

$$D_t = De^{-r(T-t)}\mathcal{N}(d_2) + V_t\mathcal{N}(-d_1)$$
$$E_t = V_t\mathcal{N}(d_1) - De^{-r(T-t)}\mathcal{N}(d_2)$$

with:

$$d_1 = \frac{\log\frac{V_t}{D} + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \quad d_2 = d_1 - \sigma\sqrt{T-t}$$

and $\mathcal{N}$ is the cumulative distribution function for a standard normal law.

3. Compute the spread value of debt. Describe qualitatively how the parameters of the model infer on the spread value.

4. We suppose that V_t and $EBITDA_t$ (Earnings Before Interest, Taxes, Depreciation and Amortization) are bounded through the following relationship: $V_t = \alpha\text{EBITDA}_t$.
We suppose that climate risk transition entails carbon pricing changes (e.g. through carbon tax). We suppose here that the firms emits θ_1 tons of CO_2 per unit of total $EBITDA$ in country 1 and θ_2 tons of CO_2 per unit of total $EBITDA$ in country 2. Carbon prices for CO_2 are now p_1 and p_2 per ton of CO_2 leading to new assets and Earnings values $\tilde{V}_t$ and $\tilde{EBITDA}_t$.
Compute the new values $\tilde{EBITDA}$ and $\tilde{V}_t$ as a function of $EBITDA$ and V_t. What is the new stochastic differential equation for $\tilde{V}_t$?

5. Considering the Debt principal remains unchanged, compute the new value of Debt $\tilde{D}_t$ and the new value of probability of default.

6. We now focus on physical risk using notations introduced at the beginning of the exercise (introduction, question 1. and 2.). Suppose that each time a climate event occurs, a fraction γ of assets value is lost.
We assume the occurrence of climate events (the number N_t of climate events that have occurred between 0 and t) is an independent Poisson process with parameter λt.
We assume that the stochastic differential equation of V_t is:

$$dV_t = (r + \gamma\lambda)V_t dt + \sigma dW_t - \gamma V_t dN_t.$$

Comment this equation.

7. Show that $\ln V_t = \ln V_0 + \sigma W_t + (r + \lambda\gamma + \frac{\sigma^2}{2})t + N_t\ln(1-\gamma)$.

8. We propose to simplify the previous equation, considering that $N_t\ln(1-\gamma)$ behaves like an independent normal variable with mean μ_t and standard deviation η_t. Express μ_t and standard deviation η_t. Compute the new default probability (Congrats! you are done with your first -simple- climate risk model).

Exercise 3: Quizz.

Select the correct answer(s).

1. In the Vasicek model, the systemic and idiosyncratic factors are:

    1. Independent but correlated random variables;
    2. Standard Gaussian random variables;
    3. Uncorrelated variables;
    4. Latent variables.

2. Which of the following statements are correct?

    1. Credit risk is mitigated and managed through diversification in retail banking;
    2. Credit risk is mitigated and managed partially through hedging in market activities and the CVA desk takes charge of it;
    3. Securitization is a way to mitigate credit risk in retail banking;
    4. CDS can be used to mitigate credit risk in retail banking.

3. Climate risk has the following impacts for banks:

    1. No or little impact;
    2. Credit risk - Transition risk (counterparties may see their business model affected by political decisions - e.g. on carbon tax);
    3. Credit risk - Physical risk (counterparties may be more frequently be affected by extreme natural events or just events - e.g. sea level, with impact on economical life, population moves, etc);
    4. Market risk (stranded assets).

4. A bank has the funding cost 50 bps and a 10% cost of capital. Compute the minimum Pricing for the following deal features: Maturity = 1 year, PD = 1%, LGD = 70%, Reg. Capital = 6% of exposure, General Expenses (internal costs) : 1% of exposure):

    1. 1.7%;
    2. 2.52%;
    3. 2.77%;
    4. 3.08%.

5. In Merton's structural model, the value of the firm's equity is equal to::

    1. The value of a call on the firm assets at maturity T with strike D (the firm's debt);
    2. The value of a put on the equity of the firm at maturity T with strike D (the firm's debt);
    3. The purchase value of risk-free zero-coupon with a facial value equal to D (the firm's debt);
    4. The expected discounted cash-flows of the shareholders.

6. What are the situation where you deem there is a wrong way risk:

    1. Buying a call option on oil to an airline company;
    2. Buying CDS protection to monolines on RMBS tranches;
    3. Buying a swap on interest rate to a bank so as to be hedged against a rise in interest rates.

7. The default probability increases with maturity. Does the spread increase with time ?

    1. Yes;
    2. No;

3. Both cases can occur (if so explain why).

8. What Initial Margin is for?

1. Mitigate settlement risk;
2. Increase colateralization requirement;
3. Reduce systemic risk.

9. Which ratings are investment grade ratings:

1. A3;
2. Baa2;
3. CCC;
4. B+;
5. AAA.

10. Which statements regarding the sensitivity of securitized tranches towards pool correlation are always correct:

1. Equity tranches benefit from higher level of correlation;
2. Equity tranches benefit from low level of correlation;
3. Mezzanine tranches benefit from higher level of correlation;
4. Mezzanine tranches benefit from low level of correlation;
5. Senior tranches benefit from higher level of correlation;
6. Senior tranches benefit from low level of correlation.