Final Test – 2022-2023

Credit Risk

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Master II

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Exercise 1: Synthetic CDO and risks correlation.

IDEA OF THE EXERCISE: combiner risque de défaut et risque de marché. Dans un CDO synthétique, le cash apporté par la vente des bonds (liabilities) est investi dans des actifs peu risqués. Il y a un risque de marché poru les porteurs de tranches (on doit vendre des actifs pour rembourser les acheteurs des CDS individuels). On peut avoir une corrélation de risques négative si on assurer par exemple des tranches de RMBS et qu'on a en mêle temps investi sur des RMBS.
In this exercise, we will consider a synthetic CDO. On the liabilities' side, the CDO is made of tranches (from equity to senior) that have been issued on the market. Theses tranches (bonds with different risks/seniorities) are invested in low risk securities and will finance the protection that is sold by the CDO on a basket of firms (e.g. 100 CDS sold on corporate names, RMBS, etc.). The securtities in which the investments have been made, as well as the variable legs of the CDS are on the assets' side.

1. Suppose there is one default on one name within the basket. Explain what happens on the CDO side and for the tranches holders.

> ....

2. So to reimburse the CDS buyers, securities have to be sold on the market, at a market price.
a. Write the mathematical relationship between both τ_{TM} and τ_{TR} and the time of cancellation (τ_C) and default (τ_D).

> The termination time of a Loan CDS is equal to the minimum date between the cancellation time and the default time, we thus have: $\tau_{TM} = \min(\tau_C, \tau_D)$.
> In order to express the triggering time of a Loan CDS, we need to use an indicator function to condition the expression of τ_{TR} on $\{\tau_D < \tau_C\}$:
> $$\tau_{TR} = \tau_D \times \mathbb{1}_{\{\tau_D<\tau_C\}} + \infty \times \mathbb{1}_{\{\tau_C\leq\tau_D\}}$$
> Note that if neither the loan is refinanced nor it defaults before maturity, we have $\tau_C = \tau_D = \infty$ and thus $\tau_{TR} = \infty$.

b. Let denote $S_C(t)$ and $S_D(t)$ the survival functions of the cancellation time and default time of a LCDS (that is $S_C(t) = \mathbb{P}(\tau_C > t)$ for instance). Show that the cumulative distribution F_{TM} and F_{TR}, respectively of the termination time and triggering time are equal to: $F_{TM}(t) = \mathbb{P}(\tau_D < t, \tau_C < t)$ and $F_{TR}(t) = \mathbb{E}(\mathbb{1}_{\{\tau_D<t\}} \times S_C(\tau_D))$.

> $$F_{TM}(t) = \mathbb{P}(\min(\tau_D, \tau_C) < t) = \mathbb{P}(\tau_D < t, \tau_C < t)$$
> $$F_{TR}(t) = \mathbb{P}(\tau_D < t, \tau_D < \tau_C) = \mathbb{E}(\mathbb{1}_{\{\tau_D<t\}} \times S_C(\tau_D))$$

3. We call s the spread of the LCDS, M the maturity of the loan, R the recovery rate on the loan, and r the constant discount rate. We assume that the nominal of the reference loan is 1. Show that the value of the Fixed Leg and the Variable Leg (FL_t and VL_t) are equal to:

$$FL_t = \int_t^M s e^{-r(h-t)} dS_{TM}(h)$$

$$VL_t = \int_t^M (1-R)e^{-r(h-t)} dS_{TR}(h)$$

> Regarding the fixed leg: the fix leg receives the LCDS premium or spread, s, that we discount, until the termination of the LCDS or the maturity of the loan.
> Regarding the variable leg: the variable leg will receive the nominal (1) times the loss given default default, $(1-R)$, that we discount, when and if a triggering event happens before maturity.

4. In this question, we assume that the time of cancellation and time of default respectively follow an exponential law of parameter c and λ and are independent. Show that the spread of the LCDS is the same as a regular CDS (same expression as in question 1).

> We have $S_{TM}(h) = \mathbb{P}(\tau_D > t, \tau_C > t) = \mathbb{P}(\tau_D > t) \times \mathbb{P}(\tau_C > t) = \exp^{-(\lambda+c)t}$.

5. We drop the independence assumption between the time of cancellation and the time of default.
a. Discuss the validity of the independence assumption made in question 4.

> It is hard to believe that the refinancing or the reimbursement of a loan is independent from the financial health and by consequence the time of default of a company. For instance, a firm with excellent financial figures and thus a low probability of default, might want to reimburse its debt. The other way round, a firm with bad financial figures might want to refinance its debt to postpone negative cash flows thus canceling the LCDS as its credit quality decreases, and thus time of default nears.

b. How would you model dependence between the cancellation time and the default time?

> Both time of cancellation and of default are of exponential law. We could model dependence using a copula, let us say a gaussian copula between the two time variables, in order to model dependence between the two random times without affecting their univariate distribution.

Exercise 2: The climate component of credit risk.

In this exercise, we will focus on the Merton model that we will adapt so as to add a climate risk component. Both questions 3 and 4 focus on transition risk impact while question 5, 6 and 7 will allow us to propose a naive model for physical risk impact on credit risk.

Notations are as follows:

- V_t is the assets' market value at time t, V_t dynamic is:

$$\frac{dV_t}{V_t} = r dt + \sigma dW_t,$$

- The firm is financed by Debt and equity,
- Debt principal is equal to D and is to be repayed at time T.

1. Recall the main assumptions of the Merton model.

> Markets are arbitrage free. Equityholders have a call on the assets, with maturity T and strike D.

2. We recall that let D be the amount of debt in the balance sheet in t, D_t its value, and E_t the value of equity (in t). We have:

$$D_t = De^{-r(T-t)}\mathcal{N}(d_2) + V_t\mathcal{N}(-d_1)$$
$$E_t = V_t\mathcal{N}(d_1) - De^{-r(T-t)}\mathcal{N}(d_2)$$

with:

$$d_1 = \frac{\log\frac{V_t}{D} + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \quad d_2 = d_1 - \sigma\sqrt{T-t}$$

and $\mathcal{N}$ is the cumulative distribution function for a standard normal law.

Compute the spread value of debt. Describe qualitatively how the parameters of the model infer on the spread value.

> We have :
>
> $$s_t = \frac{1}{T-t}\log\left(\frac{D}{D_t}\right) - r$$
>
> with $D_t = De^{-r(T-t)}\mathcal{N}(d_2) + V_t\mathcal{N}(-d_1)$ Spread increases with volatility and D. Spread decreases when T increases.

3. We suppose that V_t and $EBITDA_t$ (Earnings Before Interest, Taxes, Debt and Amortization) are bounded through the following relationship: $V_t = \alpha \mathrm{EBITDA}_t$.
We suppose that limate risk transition entails carbon pricing changes (e.g. through carbon tax). We suppose here that the firms emits θ_1 tons of CO_2 per unit of total $EBITDA$ in country 1 and θ_2 tons of CO_2 per unit of total $EBITDA$ in country 2. Carbon prices for CO_2 are now p_1 and p_2 per ton of CO_2 leading to new assets and Earnings values $\tilde{V}_t$ and $EBI\tilde{T}DA_t$.
Compute the new values $EBI\tilde{T}DA$ and $\tilde{V}_t$ as a function of $EBITDA$ and V_t. What is the new stochastic differential equation for $\tilde{V}_t$?

> $EBI\tilde{T}DA_t = (1 - \theta_1 p_1 - \theta_2 p_2)EBITDA_t$. Idem for $\tilde{V}_t$ and V_t.
> Hence the s.d.e. for $\tilde{V}_t$ remains the same as for V_t.

4. Considering the Debt principal remains unchanged, compute the new value of Debt $\tilde{D}_t$ and the new value of probability of default.

> $$D_t = De^{-r(T-t)}\mathcal{N}(\tilde{d}_2) + V_t\mathcal{N}(-\tilde{d}_1)$$
> $$E_t = V_t\mathcal{N}(\tilde{d}_1) - De^{-r(T-t)}\mathcal{N}(\tilde{d}_2)$$
>
> with:
>
> $$\tilde{d}_1 = \frac{\log\frac{\tilde{V}_t}{D} + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}} \quad \tilde{d}_2 = \tilde{d}_1 - \sigma\sqrt{T-t}$$
>
> New default probability is $\tilde{PD} = \mathcal{N}(-\tilde{d}_2) = 1 - \mathcal{N}(\tilde{d}_2)$

5. We now focus on physical risk using notations introduced at the begining of the exercise (introduction, quetion 1. and 2.). Suppose that each time a climate event occurs, a fraction γ of assets value is lost.
We assume the occurence of climate events (the number N_t of climate events that have occured between 0 and t) is an independent Poisson process with parameter λt.
We assume that the stochastic differential equation of V_t is:

$$dV_t = (r + \gamma\lambda)V_t dt + \sigma dW_t - \gamma V_t dN_t.$$

Comment this equation.

> $\gamma V_t dN_t$ is the fraction lost between dt which occurs with probability λdt hence the term λdt in the s.d.e. that is associated with a lost value γV_t. The term $\gamma V_t \lambda dt$ is therefore the expected loss value due to a climate event during dt.

Note that the expected return per dt units of time is the risk-free rate r.

6. Show that $\ln V_t = \ln V_0 + \sigma W_t + (r + \lambda\gamma + \frac{\sigma^2}{2})t + N_t \ln(1-\gamma)$.

The only "difference" with classical s.d.e. integration is the dN_t part that is integrated (quite simple integration given that the term before dN_t is constant.

7. We propose to simplify the previous equation, considering that $N_t \ln(1-\gamma)$ behaves like an independent normal variable with mean μ_t and standard deviation η_t. Express μ_t and standard deviation η_t. Compute the new default probability (Congrats! you are done with your first -simple- climate risk model).

The mean of N_t is λ_t and the variance of N_t is also λt. Hence $\mu_t = \lambda t \ln(1-\gamma)$ and $\eta_t^2 = (\ln(1-\gamma))^2 \lambda t$.
The Default Probability is

$$P\left[V_T < D\right] = P\left[\ln V_T - \ln V_0 < \ln D - \ln V_0\right] = P\left[\sigma W_T + \ln(1-\gamma)N_t < \ln D - \ln V_0 - (r + \lambda\gamma + \frac{\sigma^2}{2})T\right]$$

Hence: $PD = \mathcal{N}\left(\frac{\ln \frac{D}{V_0} - (r + \lambda\gamma + \frac{\sigma^2}{2})T - \mu_t}{(\sigma - \ln(1-\gamma)\sqrt{\lambda})\sqrt{t}}\right)$.

To calibrate the model with both transition and physical risks component, you need to:

1. *calibrate σ which is non-observable (only the equity volatility is observable)*
2. *report D as short-term debt plus a fraction of medium and long-term debt*
3. *assess p and θ (see Bouchet and Le Guenedal - see Lecture 3)*
4. *calibrate γ looking at the damages per climate event (insurers data) and λ though historics and climate scenario projections*

Note that the model can be refined so that γ is no longer fix but can be distributed along a random variable.

Exercise 3: Quizz.

Select the correct answer(s).

1. In the Vasicek model, the systemic and idiosyncratic factors are:

   1. Independent but correlated random variables;
   2. Standard Gaussian random variables;
   3. Uncorrelated variables;
   4. Latent variables.

2, 3 and 4.

2. Which of the following statements are correct?

   1. Credit risk is mitigated and managed through diversification in retail banking;
   2. Credit risk is mitigated and managed partially through hedging in market activities and the CVA desk takes charge of it;
   3. Securitization is a way to mitigate credit risk in retail banking;
   4. CDS can be used to mitigate credit risk in retail banking.

1, 2, and 3.

3. Climate risk has the following impacts for banks:

1. No or little impact;
2. Credit risk - Transition risk (counterparties may see their business model affected by political decisions - e.g. on carbon tax);
3. Credit risk - Physical risk (counterparties may be more frequently be affected by extreme natural events or just events - e.g. sea level, with impact on economical life, population moves, etc);
4. Market risk (stranded assets).

> 2, 3, and 4.

4. A bank has the funding cost 50 bps and a 10% cost of capital. Compute the minimum Pricing for the following deal features: Maturity = 1 year, PD = 1%, LGD = 70%, Reg. Capital = 6% of exposure, General Expenses (internal costs) : 1% of exposure):

1. 1.7%;
2. 2.52%;
3. 2.77%;
4. 3.08%.

> 3.

5.In Merton's structural model, the value of the firm's equity is equal to::

1. The value of a call on the firm assets at maturity T with strike D (the firm's debt);
2. The value of a put on the equity of the firm at maturity T with strike D (the firm's debt);
3. The purchase value of risk-free zero-coupon with a facial value equal to D (the firm's debt);
4. The expected discounted cash-flows of the shareholders.

> 1 and 4.

6.What are the situation where you deem there is a wrong way risk:

1. Buying a call option on oil to an airline company;
2. Buying CDS protection to monolines on RMBS tranches;
3. Buying a swap on interest rate to a bank so as to be hedged against a rise in interest rates.

> 1 and 2 (see lectures). Banks are better off with higher interest rates (better margins), hence in the third case, there is Right Way Risk.

6.In Merton's structural model, the value of the firm's equity is equal to:

1. The value of a call on the firm assets at maturity T with strike D (the firm's debt);
2. The value of a put on the equity of the firm at maturity T with strike D (the firm's debt);
3. The purchase value of risk-free zero-coupon with a facial value equal to D (the firm's debt);
4. The expected discounted cash-flows of the shareholders.

> 1 and 4.

7. The default probability increases with maturity. Does the spread increase with time ?

1. Yes;
2. No;
3. Both cases can occur (if so explain why).

> Spread increases with maturity for assets with a good credit quality (they "only" can downgrade their credit quality over time) while it gets lower for low quality assets (the credit risk is anticipated to occur during the first years).

8. What Initial Margin is for?

   1. Mitigate settlement risk;
   2. Increase colateralization requirement;
   3. Reduce systemic risk.

> 1 and 4.

9. Which ratings are investment grade ratings:

   1. A3;
   2. Baa2;
   3. CCC;
   4. B+;
   5. AAA.

> 1, 2 and 5.

10. Which statements regarding the sensitivity of securitized tranches towards pool correlation are always correct:

    1. Equity tranches benefit from higher level of correlation;
    2. Equity tranches benefit from low level of correlation;
    3. Mezzanine tranches benefit from higher level of correlation;
    4. Mezzanine tranches benefit from low level of correlation;
    5. Senior tranches benefit from higher level of correlation;
    6. Senior tranches benefit from low level of correlation.

> Equity loss is a put on pool losses: hence 1 is true. Senior tranches loss is a call on pool losses: hence 6 is true. Impossible to say for mezzanine tranches.