Credit Risk

Lecture 1 – Introduction, reduced-form models and CDS

Loïc BRIN

École Nationale des Ponts et Chaussées
Département Ingénierie Mathématique et Informatique (IMI) – Master II
1. Class structure and assignments

2. Credit risk management is at the base of our economies

3. Main credit risk modeling outcomes and challenges

4. The basics of credit risk

5. Reduced-form models

6. Single-name credit derivatives and Credit Default Swaps (CDS)
1 - Class structure and assignments
All the information is on the website:

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defaultrisk.free.fr
```

This course is part of the training cycle of École Nationale des Ponts et Chaussées: it is a Master II course from the Département Ingénierie Mathématique et Informatique (IMI).

Teaching team, syllabus, grading, access to the forum, etc.
Class structure and assignments
Pedagogical tools (I/II)

**Theory**
- Infography
- Cheat Sheet
- Slides
- References

**Practice**
- Quizzes/Flipcards
- Articles and papers
- R/Python notebooks
- Tutorial
- Project
- Case Study

**Communication**
- Forum
- Questions in class
- Office Hours

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Credit Risk - Lecture 1
Class structure and assignments
Pedagogical tools (II/II)

Buttons
- Quiz
- R Markdown
- Tutorial
- Newspapers
- Be Careful!
- Theorem
- Definition

Blocks
- Definition - Math
  I use this block for math definitions.
- Definition - Eco or finance
  I use this one for economics or finance definition.

Example
I use this one when giving concrete examples.

Other styles
- This is a proof.
- This is the solution of an exercise, or details of an explanation.

I am citing: [Harrison and Kreps, 1979].

I will emphasize important words.
Objectives of the lecture

At the end of this lecture, you will:

▶ Understand why credit risk is at the basis of our economies;
▶ Have a clear view on the credit risk modeling challenges and outcomes;
▶ Know the basic concepts of credit risk, that is, how to price a bond, what a spread is and how to extract it from the price of a bond, what are the Exposure At Default (EAD), Loss Given Default (LGD) and Probability of Default (PD);
▶ Know what reduced-form models are and how to calibrate them;
▶ Know what Credit Default Swaps are and how to price them.
2 - Credit risk and economics
Credit risk management is at the base of our economies

What is credit risk?

*credo*: I believe (latin)

*resecare*: To break (latin)

---

**Credit risk – Definition**

Credit risk is the risk of **default** on a debt, that may arise from a borrower failing to make required payments. [BCBS, 2000]

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[Fergusson, 2008] is an interesting reference to tackle the subject from an historical perspective.
Credit risk management is at the base of our economies

Why is there credit risk?

There is a **discrepancy of financial needs** among economic agents. Some agents need money to fulfill their projects (firms, states, people, etc.) and other do not need an immediate access to their wealth.

To fill this gap, lenders lend to borrowers, based on the **belief** that they will retrieve their money.

This belief – this trust – is at the origin of credit risk.
Credit risk management is at the base of our economies

Who finance the economy?

| Source: Aspects of Global Asset Allocation, IMF. and personal cross-checkings. |

<table>
<thead>
<tr>
<th>Banks</th>
<th>Mutual funds and Insurance companies</th>
<th>Private Wealth</th>
</tr>
</thead>
<tbody>
<tr>
<td>130 TUSD</td>
<td>58 TUSD</td>
<td>37 TUSD</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Pension funds</th>
<th>Foreign Exchange Reserve</th>
<th>Sovereign Wealth Funds</th>
</tr>
</thead>
<tbody>
<tr>
<td>34 TUSD</td>
<td>11.7 TUSD</td>
<td>7.3 TUSD</td>
</tr>
</tbody>
</table>
Credit risk management is at the base of our economies

Who borrow?

- **Corporate bonds**: 86 TUSD
- **Equity**: 69 TUSD
- **Loans**: 76 TUSD
- **Public debt securities**: 58 TUSD

**Source**: The Random Walk, Mapping the world financial markets, 2014, DB research.
Credit risk management is at the base of our economies

In what banks differ from other lenders?

They have an expertise and a defined economic purpose as financial intermediaries.

- They have an expertise in **maturity transformation** (ALM\(^1\) department);
- They have much more **information** on the economy and on their counterparties than any other agent;
- They know how to dissociate risks and underlying assets thanks to **derivative products**;
- They can deal with credit risk on a **macro level** (portfolio approach, dynamic management of assets, macro hedging strategy);
- They **create money** when allowing credits.

See several references at the end of the slides.

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\(^1\) Asset Liability Management.
Credit risk management is at the base of our economies

Regulatory requirements for banks

Banks are at the center of our economies
- Finance the economy;
- Provide way to transfer risks;
- At the basis of money creation.

Banks face numerous and complex risks
- Credit Risk (~80 %*);
- Market Risk (~10 %*);
- Liquidity Risk;
- Operational Risk (~10 %*).

* Computed as a percentage of Risk Weighted Assets (RWA, see Lecture 5).

It is thus a highly regulated sector
Banks must set capital apart to face an unlikely rise of these risks.

Incentives to reduce balance sheet size

Moving towards an originate to distribute model ? (vs originate to hold)
Conclusion
Credit risk management is at the base of our economies

- Credit risk is the risk that a borrower fails to make required payments;
- There is no financing of the economy without credit risk;
- Banks finance a big chunk of the economy and are thus prone to credit risk.
3 - Credit risk outcomes
The outcomes we will face in this class

Very different outcomes in comparison with market risk

<table>
<thead>
<tr>
<th>Market Risk</th>
<th>Credit Risk</th>
</tr>
</thead>
<tbody>
<tr>
<td>Amount of data</td>
<td>A lot</td>
</tr>
<tr>
<td>Liquidity of the assets</td>
<td>Liquid</td>
</tr>
<tr>
<td>Shape of the loss function</td>
<td>Symmetric</td>
</tr>
<tr>
<td>Correlations</td>
<td>High</td>
</tr>
<tr>
<td>Risk Management</td>
<td>Hedging</td>
</tr>
<tr>
<td>Backtesting</td>
<td>Possible</td>
</tr>
</tbody>
</table>
### The outcomes we will face in this class

- **Point in time**
  - Through the Cycle

- **Time horizon**
  - One Year
  - Several Years

- **Accountancy**
  - In balance-sheet
  - Off balance-sheet

- **Number of counterparties**
  - Single-name models
  - Portfolio models

- **Assets**
  - With assets
  - Without assets

- **What to predict?**
  - Two-state model
  - Continuous model

- **Purpose**
  - Regulatory purpose
  - Internal purpose

- **Probability**
  - Risk Neutral Probability
  - Real World Probability

- **Data**
  - Lack of data
  - Lot of data

Let us take a closer look at the two latter.
Real World and Risk Neutral probability

Real World and Risk Neutral probability – Definitions

Let $S_t$ be the variable equals to $s$, if the future state of the economy, in $t$, is $s$.

**Real World Probability, $P$**

Probability that an event, $s$, occurs.

**Risk Neutral Probability, $Q$**

Probability measure which weights the future state of the economy, $s$, according to the price to be risk neutral to that specific state, proportionally to the price to be risk neutral to all the future states of the economy.

This formalization was made by [Harrison and Kreps, 1979].
Real World and Risk Neutral probability

A simplified example to understand Risk Neutral Probability

A simplified example – Real World and Risk neutral probability

Let us say that a future state of the economy (in 3 years) will occur with a (Real World) probability of 10 %. Let us suppose that:

- to be sure (i.e. to be risk neutral) to have a cash flow of 1 EUR in 3 years (that is the price of a risk-free zero-coupon bond) costs 0.8 EUR;
- to be sure (i.e. to be risk neutral) to have a cash flow of 1 EUR, only if, the specific state $s$ of the economy occurs in 3 years, costs 0.1 EUR.

We have that: $Q(S_t = s) = \frac{0.1}{0.8} = 12.5 \%$ even if $P(S_t = s) = 10 \%$.

In that case, the cost of protecting oneself against $s$ is higher than suggested by the Real World probability.
Real World and Risk Neutral probability

Risk Neutral Default rates vs Real World Default rates

<table>
<thead>
<tr>
<th>Rating (in %)</th>
<th>Real World Default rate</th>
<th>Risk Neutral Default rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>0.03</td>
<td>0.60</td>
</tr>
<tr>
<td>AA</td>
<td>0.06</td>
<td>0.73</td>
</tr>
<tr>
<td>A</td>
<td>0.18</td>
<td>1.15</td>
</tr>
<tr>
<td>BBB</td>
<td>0.44</td>
<td>2.13</td>
</tr>
<tr>
<td>BB</td>
<td>2.23</td>
<td>4.67</td>
</tr>
<tr>
<td>B</td>
<td>6.09</td>
<td>8.02</td>
</tr>
<tr>
<td>CCC</td>
<td>13.52</td>
<td>18.39</td>
</tr>
</tbody>
</table>

Risk Neutral Default rates are higher than Real World Default rates.

That could be because of:
- The lack of liquidity on the debt market;
- The lack of information of investors on the market;
- The risk aversion of investors on the market, etc.
Credit risk modeling and the challenge of data

Where to find data for credit risk modeling?

- Data to estimate with real world probability
  - Financial Reports
  - Rating agencies
  - Banks knowledge

- Data to estimate with risk neutral probability
  - Financial markets prices (bonds, equity, CDS, etc.)
Conclusion
Main credit risk modeling outcomes and challenges

- Contrary to market risk, credit risk faces the following challenges: there is few data, the market is illiquid, the loss functions are asymmetric, correlations are low and backtesting is hardly possible;

- Additionally, credit risk can be envisioned in many different ways: on several time spans, with a real-world or risk neutral approach, in a continuous or binary perspective, etc. making this risk particularly technical.
4 - Credit risk: The basics
Pricing of bonds – Continuous version

The spread of a bond

Pricing a bond – Continuous version

Let us denote the continuous and constant coupon rate by $c$, the risky rate of the firm $A$ by $r^A$, the maturity of the bond $T$, and assume the nominal, $N$, is equal to 1, the price of a bond is:

$$\tilde{B}^A(0, T) = 1 + (c - r^A) \frac{1 - e^{-r^A T}}{r^A}$$

The formula is a simple result consequent to the no-arbitrage assumption and the integral resolution of:

$$\tilde{B}^A(0, T) = \int_0^T c e^{-r^A t} dt + Ne^{-r^A T}$$

The spread of a bond – Continuous version

For a given risk-free rate, $r$, the spread of a bond of price $\tilde{B}^A(0, T)$, is the value $s^A$ so that:

$$\tilde{B}^A(0, T) = 1 + (c - (r + s^A)) \frac{1 - e^{-(r+s^A) T}}{(r + s^A)}$$
The implied survival probability – Computed with prices

Let $\tau$ be the time of default of firm A. Let $\bar{B}^A(t, T)$ be the price of a zero-coupon risky bond of firm A, at $t$, of maturity $T$, and nominal $N$. Let $B(t, T)$ be the price of a zero-coupon risk-free bond, at $t$, of maturity $T$, and nominal $N$.

The implied survival probability of firm A, in $T$, from $t$, is:

$$Q(\tau > T \mid \tau > t) = \frac{\bar{B}^A(t, T)}{B(t, T)}$$

It is a consequence of the no-arbitrage assumption.

The implied survival probability – Computed with constant continuous spreads

Let $s^A$ be the spread of Firm A. The implied survival probability can be written:

$$Q(\tau > T \mid \tau > t) = e^{-s^A(T-t)}$$

$$Q(T > \tau \mid \tau > t) = \frac{Ne^{-r(T-t)}}{Ne^{-(r+s^A)(T-t)}} = e^{-s^A(T-t)}$$
Three points of attention – Discrete version, recovery and risk-free rate

Continous vs Discrete (I/IV)

Pricing a bond – Discrete version

Let \( N \) be the nominal of a bond from firm \( A \), \( t_1, \ldots, t_n = T \) the dates when the coupons \( C \) are paid, \( r_1^A, \ldots, r_n^A \) the respective risky rates and, \( T \), its maturity. The price of the bond \( \bar{B}^A(0, T) \), in \( 0 \), is:

\[
\bar{B}^A(0, T) = \sum_{i=1}^{n} \frac{C}{(1 + r_i^A)t_i} + \frac{N}{(1 + r_T^A)^T}
\]

The spread of a bond – Discrete version

Let \( \bar{B}^A(0, T) \) be the price of a bond and \( r_1, \ldots, r_n \) the risk-free rates in \( t_1, \ldots, t_n \). The spread of \( A \) is \( s^A \) so that:

\[
\bar{B}^A(0, T) = \sum_{i=1}^{n} \frac{C}{(1 + r_i + s^A)t_i} + \frac{N}{(1 + r_n + s^A)^T}
\]
Bootstrap technique to compute implied survival probability – Discrete version

Suppose firm A has \( n \) bonds, \( B_1^A, \ldots, B_n^A \), paying coupons, \( C \), in \( t_1, \ldots, t_n \) and of maturity, respectively \( t_1, \ldots, t_n \). Bootstrap works the following way:

- With \( B_1^A \), it is possible to compute \( r_1^A \);
- With \( B_2^A \), substracting its cash-flow in \( t_1 \) discounted by \( 1 + r_1^A \), it is possible to compute \( r_2^A \);
- etc.

That way, one can compute iteratively \( r_1^A, r_2^A, \ldots, r_n^A \) and thus deduce \( s_1^A, s_2^A, \ldots, s_n^A \) using the risk-free rates.
Three points of attention – Discrete version, recovery and risk-free rate

Continous vs Discrete (III/IV)

Bond clean and dirty prices

Discrete payments implies discontinuity in bond valuation each time a coupon is paid: these discontinuous prices are called dirty prices. On the markets, the prices do not suffer such a problem as the so-called clean prices are quoted. The formula that links both is:

$$\text{Dirty price} = \text{Clean price} + \text{Accrued interests}$$

where

$$\text{Accrued interests} = \frac{\# \text{ days since last coupon}}{\# \text{ days between coupons}} \times \text{Coupon rate} \times \text{Nominal}$$
Adobe System Inc. bond valuation

Adobe Systems Inc. (NASDAQ: ADBE) has 600 MUSD worth of bond payable outstanding. The 1 000 USD par, 3.25 % semi-annual coupon bonds are due to mature on 1st February 2015. The coupon dates are 1st February and 1st August. They follow 30/360 day count convention and next coupon is due on 1st August 2013. Yvonne Barnet bought 1 000 such bonds from Charles Schwab on 20th July 2013. The market requires buyer to compensate seller for the accrued interest. How much Yvonne must pay Charles? Yvonne must pay the dirty price, but she only knows the clean price: 1036.10 USD.

- days between the transaction date and next coupon date = 11 = 10 days of July plus 1 day of August;
- days in the coupon period = 180 (since 30/360 day count convention is used).

Thus, the dirty price is: $1036.10 + \frac{1000 \times 3.25\%}{2} \times \frac{169}{180} = 1051.36$ USD
Three points of attention – Discrete version, recovery and risk-free rate

The importance of the recovery rate

To simplify math formulas, the recovery rate – the extent to which principal and accrued interests on defaulted debt can be recovered, expressed as a percentage of face value – is often forgotten (or equivalently supposed equal to one). Nonetheless, in practice, the recovery rate must be taken into account when extracting the probability of default from a market price.

The implied probability of default taking into account recovery

Let $R$ be the recovery rate, the implied probability of default taking into account recovery is:

$$ PD = \frac{1 - \frac{\bar{B}(0, T)}{B(0, T)}}{1 - R} $$

This is a consequence of the no-arbitrage hypothesis.
Three points of attention – Discrete version, recovery and risk-free rate

What is the risk-free rate?

The rates used as a risk-free rates evolved during the last decades:

- from the government rates, first;
- to the LIBOR rates, then;
- to the Overnight Indexed Swap (OIS) rate, now.
The general framework of credit risk modeling: PD, EAD and LGD

Definitions of PD and EAD

**Probability of Default – PD**

It is the *one-year default probability* of a counterparty.

**Exposure At Default – EAD**

Exposure At Default is the *loss* a bank would suffer if its counterparty defaults, and there would be no guarantee.

This value can either be *known* (loans for example) or *unknown* (lines of credit for instance).
The general framework of credit risk modeling: PD, EAD and LGD

Focus on the Loss Given Default (I/II)

Loss Given Default – LGD

Let $R$ be the recovery rate, that is the percentage of the exposure recovered by the bank after the default, we have:

$$LGD = 1 - R$$

Loss Given Default and $R$ depend on the underlying credit contract

<table>
<thead>
<tr>
<th>Contracts</th>
<th>$R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bank loans</td>
<td>80.3 %</td>
</tr>
<tr>
<td>Senior secured bonds</td>
<td>63.5 %</td>
</tr>
<tr>
<td>Senior unsecured bonds</td>
<td>49.2 %</td>
</tr>
<tr>
<td>Senior subordinated bonds</td>
<td>29.4 %</td>
</tr>
<tr>
<td>Subordinated bonds</td>
<td>29.5 %</td>
</tr>
<tr>
<td>Junior subordinated</td>
<td>18.4 %</td>
</tr>
</tbody>
</table>

Source: Moody’s statistics.
Modeling LGD with a beta distribution using a Maximum Likelihood estimator

Let $\alpha > 0$, $\beta > 0$. The density of a beta distribution is:

$$f(x; \alpha, \beta) = \begin{cases} 
\frac{x^{\alpha-1}(1-x)^{\beta-1}}{\int_0^1 u^{\alpha-1}(1-u)^{\beta-1} \, du} & \text{for } x \in [0, 1] \\
0 & \text{else}
\end{cases}$$

From data, it appears that the shape of LGD distributions is usually a **U-shaped** curve.
The general framework of credit risk modeling: PD, EAD and LGD

The Expected Loss – EL

We define the Expected Loss as:

\[
EL = \mathbb{E}(EAD \times 1\{\tau < M\} \times LGD)
\]

\[
= \mathbb{E}(EAD) \times \mathbb{E}(1\{\tau < M\}) \times \mathbb{E}(LGD)
\]

Be Careful!

The independence of EAD, PD and LGD

There is no reason why one should assume independence between EAD, PD and LGD. Actually, some phenomena and papers proved the contrary:

▶ The phenomenon of gambling for resurrection;
▶ The study by [Frye, 2013] postulates that LGD is a function of the probability of default.

Quiz

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Credit Risk - Lecture 1
Conclusion
The basics of credit risk

- Pricing of bonds, in a continuous or discrete framework, is based on the **no-arbitrage assumption**;
- **Coupons and recovery** are complexities that need to be taken into account when implying probabilities of default from bond prices through respectively accrued interests cleaning and bootstrapping;
- There are **three key components** of any credit expected losses estimator, the Exposure At Default, the Loss Given Default and the Probability of Default – their independence, while often assumed in models, is not always tenable;
- The **Probability of Default (PD)** is the major focus of this course.
## 5 - Reduced-form models
Reduced-form models

**Definition**

Reduced-form models consist in modeling the conditional law of the random time of default.

**Conditional default probability**

In a reduced-form model, conditional default probability is defined as:

\[ Q(\tau < t + dt \mid \tau > t) = \lambda dt \]

\( \lambda \) is the default intensity and corresponds to the instantaneous forward default rate. This variable is exogenous to the problem.

**Survival function**

The survival function is thus defined as:

\[ S(t) = Q(\tau > t) = \exp(-\lambda t) \]
Reduced-form models

What is default intensity?

Is the default intensity, $\lambda$, constant or stochastic?

It depends:

- **Constant**: Time homogeneous Poisson Process;
- **Deterministic**: Time deterministic inhomogeneous Poisson Process;
- **Stochastic**: Time-varying and stochastic Poisson Process as the Cox, Ingersoll, Ross (CIR) model.
Reduced-form models
Calibration of default intensity models

The implied survival probability

Let $B(0, t)$ be a zero-coupon risk-free bond and $\bar{B}(0, t)$ be a risky zero-coupon bond. We have:

$$Q(\tau > t) = \frac{\bar{B}(0, t)}{B(0, t)}$$

This is a result based on the no-arbitrage assumption.

The implied survival probability – Application for calibration of intensity models

We deduce from the above formula the expression of the default intensity, $\lambda$:

$$\lambda = -\frac{\log \left( \frac{\bar{B}(0, t)}{B(0, t)} \right) }{t}$$

Be Careful!

We have seen that, $Q(\tau > t) = e^{-\lambda t}$
Reduced-form models

Conclusion

Reduced-form models are models based on an exogenous variable, called here, the default intensity;

- The default intensity is often assumed constant but can also be non-constant or even stochastic.
6 - Single-name derivatives
Single-name credit derivatives and Credit Default Swap (CDS)

Credit Default Swap

Credit Default Swaps (CDS) are financial agreements that allows the transfer of the credit risk of a loan to another counterparty.

- Bank A holds a loan on corporate C on its balance-sheet and buys protection on the credit risk related to counterparty C;
- Bank B sells protection on corporate C and receives the payment of a premium.

Are CDS insurance contracts?

CDS are neither insurance contracts, nor guarantees.
Single-name credit derivatives and Credit Default Swap (CDS)

CDS cash flows
Statistics on the CDS market

The derivatives market

The CDS market

10.0 TUSD (notional)
Legal aspect of CDS

The International Swaps and Derivatives Association (ISDA)

ISDA defines **standards** for credit derivatives transactions:

- **1999**: definitions;
- **2003**: supplements;
- **2009**: big-bang protocol.

Examples of important specifications in a CDS

- **What is a CDS credit event?**
  - bankruptcy;
  - failure to pay (coupon or nominal);
  - restructuring of a bond.

- **Settlement:**
  - Physical settlement: the buyer of the CDS gives the defaulted bonds to the seller, and receives the nominal \( N \);
  - Cash settlement: the buyer receives \( N(1 - R) \), and keeps the defaulted bonds.

- **Normal vs Digital CDS:**
  - Digital: the payoff is \( N \);
  - Normal: the payoff is \( (1 - R)N \).
Single-name credit derivatives and Credit Default Swap (CDS)

Use of CDS

What CDS can be used for? Risk management and investment strategies.

- Risk mitigation;
- Management of credit lines;
- Bank’s capital management;
- Balance sheet management;
- Leverage effect (FtD, CDO);
- Access to the market.

Who uses CDS?

- Banks;
- Corporates;
- Insurers and reinsurers;
- Asset managers;
- Hedge funds.
Value of the fixed leg of a CDS

The value of the fixed leg of a CDS is:

\[
\text{Fixed}(0, T) = s(0, T) \frac{1 - e^{-(r+\lambda)T}}{r + \lambda}
\]

Fixed leg pays the reference spread at inception of the CDS up to the minimum of the default date \((\tau)\) and maturity \(T\).

For a continuous paid spread:

\[
\text{Fixed}(0, T) = \mathbb{E}^Q \left( s(0, T) \int_0^{\tau \wedge T} e^{-rt} \, dt \right)
= s(0, T) \int_0^T e^{-(r+\lambda)t} \, dt
= s(0, T) \frac{1 - e^{-(r+\lambda)T}}{r + \lambda}
\]
Value of the floating leg of a CDS

The value of the floating leg of a CDS is:

\[
\text{Floating}(0, T) = (1 - R) \frac{\lambda}{\lambda + r} \left(1 - e^{-(\lambda + r)T}\right)
\]

The floating leg is paid when a default occurs: the protection seller pays the difference between the notional and the recovery.

\[
\text{Floating}(0, T) = \mathbb{E}^Q \left((1 - R)e^{-r\tau} \mathbb{1}_{\{\tau < T\}}\right)
\]

\[
= (1 - R) \frac{\lambda}{\lambda + r} \left(1 - e^{-(\lambda + r)T}\right)
\]
Reduced-form models applied to CDS pricing

Spread of a CDS, its Mark-to-Market value and time sensitivity

The spread of a CDS

The **spread of a CDS** is:

\[ s = \lambda (1 - R) \]

At inception, the Net Present Value (NPV) for the protection seller is:

\[ \text{MtM}(0, T) = \text{Fixed}(0, T) - \text{Floating}(0, T) \]

The fair spread sets the initial MtM at 0. We thus have \( s = \lambda (1 - R) \).

MtM through the time and sensitivity of the CDS to the time

The sensitivity of the MtM is the **risky duration**, \( \text{DV} \):

\[ \text{DV}(t, T, \lambda) = \frac{1 - e^{-(r+\lambda)(T-t)}}{r + \lambda} \]
Reduced-form models applied to CDS pricing

CDS sensitivity

**Present Value of a CDS through the time**

Let $s_0$, be the spread in $t = 0$, and $s$, the spread today, in $t$. The **Present Value of the protection seller** is:

$$PV(s_0, s_t) = DV(0, t, \lambda)(s_0 - s_t)$$

- At inception, the Net Present Value (NPV) for the protection seller is: $MtM(0, T) = 0$.
- Day 2: the market spread moves to $s$. The PV of the fixed leg does not change. The PV of the floating leg changes.
- If the spread increases, credit risk increases and then, the PV of the floating leg increases.
- A CDS issued day 2, would have equal floating and fixed legs, with fair spread equal to market spread $s$.
- The PV of the protection seller is then: $DV(0, t, \lambda)(s_0 - s)$. 
Reduced-form models applied to CDS pricing

CDS – Discrete vs continuous pricing

Once again, these formulas suppose continuous interest rates payments and spread payments. These formulas are nice proxies to assess CDS spreads, but for precise pricing, one must take into account each flow separately.
Reduced-form models applied to CDS pricing

CDS spread vs Bond spread: the CDS basis

CDS spread and Bond spread

Generally, CDS spreads are larger than Bond spreads.

Several reasons explain this phenomenon:

▶ definition of credit event is different;
▶ the protection buyer has no impact on the bond issuer through covenants.

On the other hand, funds and insurers sell massively protection, making CDS spread tighten.

For more information on the subject, you can take a look at [Choudhry, 2006].
Default swaptions and other swaps

An introduction to default swaptions

Knock-out swaptions

Knock-out swaptions:

- Allow to sell or buy protection at maturity;
- Payoff is null if default occurs before maturity.

The pricing is done with models similar to Black-Scholes model.
Default swaptions and other swaps

Other swaps

- **Constant Maturity Credit Default Swaps (CMCDS):** is a usual CDS except that the *fixed* leg of the contract is computed each semester with the new data. Its *pay-off* is often capped.

- **Loan-CDS (LCDS):** they are linked to a specific loan, not on a specific name; thus, would the loan be reimbursed in advance, the CDS would be cancelled.

- **Forward CDS:** they are CDS that offer protection from a future date, \( T \) to a future data \( T + M \).

- **Cancellable CDS:** they are CDS that are cancellable at a or several future dates;

- **Total Return Swap (TRS):** they are financial contracts that transfer both the credit and the market risk of an asset against either a variable or a fixed rate (they are in fact like funded Interest Rate Swaps – IRS).
Conclusion

Reduced-form models

- CDS are contacts that **protect against default**;
- Their valuations reconcile the **no-arbitrage assumption and the reduced-form models**;
- CDS spreads **can differ from bond spreads**.

▶ Quiz
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