

## Tutorial 3

# Structural models

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### Exercise 1: Term structure of spreads in Merton's model.

1. Write the value of the debt of the firm for the debt-holders, of the shares of the firm for the shareholders, as an option on the value of the firm and with maturity the maturity of the debt.
2. Prove the Call-Put parity in that case.
3. Give the formula of the price of the shares and the one of the debt of the firm.
4. Compute the sensitivity of the price of the share with all the parameters of the problem.
5. Compute the term structure of spreads in Merton's model and comment the results.

### Exercise 2: Leland's model (1994).

In the following exercise, the underlying model is Leland's, in its easiest form (1994).

Let us consider a firm which asset value follows, under the historical probability, the following diffusion:

$$\frac{dA_t}{A_t} = (\mu - \delta)dt + \sigma dW_t$$

where  $W_t$  is a standard brownian motion.

The firm is financed through debt and equity that are exchanged on financial markets. Debt has a simple structure: the nominal is  $D$ , it was borrowed at  $t = 0$  until the end of time, and the firm has to pay a coupon,  $C$ , until the end of time.

The firm is managed by the shareholders. These can, at any moment, decide to stop the activity and trigger the bankruptcy.

$r$  is the risk free rate.

1. In the case the firm will never default, what is the value of the debt?
2. We recall that the Laplace transform  $L(a, b, \mu) = \Phi_{\tau_B}(a)$  for  $\tau_B = \inf\{t \mid W_t + \mu t \geq b\}$  is given by:

$$\Phi_{\tau_B} = \mathbb{E}[e^{-a\tau_B}] = e^{b(\mu - \sqrt{\mu^2 + 2a})}$$

. Since  $A_t$  is a Geometric Brownian Motion we recall that:

$$A_t = \exp\left(\ln(A_0) + \left(r - \delta - \frac{1}{2}\sigma^2\right)t + \sigma W_t\right)$$

Compute the value for  $D_0$ .

Paying the debt induces a fiscal rebate of  $\tau C$  for each period of time, where  $\tau$  is the abatement rate. Furthermore, a fraction  $\alpha$  of the assets is lost as bankruptcy costs in case of failure of the firm.

3. Compute the market value of the fiscal abatement, and its bankruptcy costs.

4. What is the value of a zero-coupon that pays 1 EUR in case of failure of the firm.

Adding to the assets, the value of the fiscal abatement and deducting the costs of bankruptcy, we get the total market firm value.

5. Give the total market value of the firm.

6. Debt was sold at par, that is  $P = \frac{C}{r}$ . Compute the market value of the firm.

7. Compute the market value of the shares.