Tutorial 1

Introduction, reduced-form models and CDS

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The first two exercises are taken from Options, futures, and other derivatives, *Hull C. You can find other exercises of this kind in the book. An Excel version of the correction is available here:* `http://defaultrisk.free.fr/tutorials/ENPC_CreditRisk_Lecture1_Tutorial.xlsx`.

Exercise 1: Implied probability of default and discrete payment of coupons.

Let us suppose that the LIBOR/swap rate curve (risk free rate) is flat equal to 3% (compound rate), and that a 4-year corporate bond provides a coupon of 4% per year payable semiannually and it has a yield of 5% expressed with continuous compounding. Assume that defaults can take place at the end of the year (immediately before a coupon or principal payment) and that the recovery rate is 30%. Estimate the risk-neutral default probability on the assumption that it is the same each year.

First, we want to find each year expected loss expressed as a risk-neutral valuation of the future cash flows. For this matter, let us fill a table with for each year:

- PD: the risk neutral probability of default, always equal to Q as assumed in this exercise;
- Recovery Rate: equal to 30%;
- Risk-neutral value: which is the value of the bond, as seen at the beginning of each year, and valued as the risk-free discounted future cash flows;
- Loss Given Default (LGD): Recovery × Risk-neutral value;
- Discount rate: the risk-free discount rate;
- Expected loss: LGD × Discount rate.

Date	PD	Recovery	Risk-free value	LGD	Discount rate	Expected loss
1	Q	30	104.78	74.78	0.9704	72.57Q
2	Q	30	103.88	73.88	0.9418	69.58Q
3	Q	30	102.96	72.96	0.9139	66.68Q
4	Q	30	102.00	72.00	0.8869	63.86Q
Total						**272,69Q**

Given the no arbitrage assumption, the sum of the expected losses must be equal to the difference between the risky bond and its equivalent risk-free bond.

The obligation provides a 2% of the nominal coupon every 6 months (4% annually) and its yield is 5%. It's market price is then 96.19.

The equivalent risk-free bond's value is computed discounting the expected cash-flows at the 3% rate, which gives 103.66.

The total expected loss in case of a default is thus 103.66-96.19 = 7.46. The implicit value of Q thus verifies 272.69Q=7.46, that is Q=0.0274.

Exercise 2: CDS pricing and discrete payment of spreads.

Suppose that the risk-free zero-coupon curve is flat at 7% per annum with continuous compounding and that defaults can occur halfway through each year in a new 5-year credit default swap (CDS). We assume that in case of default, the protection is paid at the end of the year, and that half of the annual spread is paid in case of default during the second semester of the year. Suppose that the recovery rate is 30% and the default probability each year conditional on no earlier default is 3%.

1. Estimate the credit default swap spread.

- Fix leg - End of year payment

Year	PD	Survival probability	Discounting	Discounted expected payments
1	0,0300	0,9700	0,9324	0,9044s
2	0,0291	0,9409	0,8694	0,8180s
3	0,0282	0,9127	0,8106	0,7398s
4	0,0274	0,8853	0,7558	0,6691s
5	0,0266	0,8587	0,7047	0,6051s

Thus, the value of the fix leg, end of year payment, is 3,7364s.

- Fix leg - Mid-year payment

Year	PD	Discounting	Discounted expected payments
0,5	0,0300	0,9656	0,0145s
1,5	0,0291	0,9003	0,0131s
2,5	0,0282	0,8395	0,0118s
3,5	0,0274	0,7827	0,0107s
4,5	0,0266	0,7298	0,0097s

Thus, the value of the fix leg, mid-year payment, is 0,0598s.

- Variable leg - Mid-year year payment

Year	PD	Recovery rate	Discounting	Discounted expected payments
0,5	0,0300	0,3000	0,9656	0,0203
1,5	0,0291	0,3000	0,9003	0,0183
2,5	0,0282	0,3000	0,8395	0,0166
3,5	0,0274	0,3000	0,7827	0,0150
4,5	0,0266	0,3000	0,7298	0,0136

Thus, the value of the variable leg, mid-year payment, is 0,0838.

- Variable leg - End of year payment

There is no flow of the variable leg that occurs at the end of the year in this exercise.

The no-arbitrage assumption states that the value of the fixed leg equals the one of the value of the variable leg. Thus, we have that: $3,7364s + 0,0598s = 0,0838$, and the spread of the CDS is 221 bps.

2. What is the value of the swap per dollar of notional principal to the protection buyer if the credit default swap spread is 150 basis points?

If the spread is 150 bps, the value of the CDS for the buyer of protection is : $0,0838-(3,7364+0,0598)\times 0,0150$. The value of the CDS is 268 bps of the nominal.

3. What is the credit default swap spread if it is a binary CDS?

If the CDS is digital, we have to adjust the variable leg mid-year payment valuation as follows:

Year	PD	Discounting	Discounted expected payments
0,5	0,0300	0,9656	0,0290
1,5	0,0291	0,9003	0,0262
2,5	0,0282	0,8395	0,0237
3,5	0,0274	0,7827	0,0214
4,5	0,0266	0,7298	0,0194

Thus, the value of the variable leg, mid-year payment, is 0,1197.

The no-arbitrage assumption states that value of the fixed leg equals the one of value of the variable leg. Thus, we have that: $3,7364s+0,0598s=0,1197$, and the spread of the CDS is 315 bps.

Exercise 3: Position on the spread curve.

In this exercise, we assume that all the flows are continuous.

1. Let us consider a seller of protection for 5 years on firm A. In case of default of A, the recovery rate is of 60%. Knowing that the spread is 70 bps, what is the default intensity?

We have that: $\lambda = \frac{s}{1-R}$, thus $\lambda = 1.75\%$.

2. What is the sensitivity of the short position price to the spread? Let us take $r = 4\%$.

We have that:

$$DV = \frac{1-e^{-(r+\lambda)T}}{r+\lambda}$$

We thus have that the sensitivity of the short position to the spread, $-DV$, is of 4.35 meaning that for an increase of 1 bp of the spread, the seller of the protection suffers a decrease of 4.35 bps of the mark-to-market (MTM) value of his CDS.

3. What is, considering constant the default intensity, the seven year spread?

If we consider the default intensity constant, we then have that the seven year spread is the same as the five year spread, that is: 70 bps.

4. The spread curve is expected to steepen. Between a five-year and a seven-year CDS, which one the trader should sell, knowing that he wants to be insensitive to any parallel shifts of the spreads?

As the spread curve is supposed to steepen, one wants to sell the five year protection and buy the seven year protection. In order to be insensitive to any parallel shift of the spread curve, the trader wants to find A_5 and A_7, respectively the nominal of the 5 and 7 year protection to buy, following the equation:

$$A_5 \times \delta_s \times DV_5 + A_7 \times \delta_s \times DV_7 = 0$$

So for example, would he sells 15 MEUR of nominal of the five-year protection, he would buy $15 \times \frac{DV_5}{DV_7} = 11.3$ MEUR of the 7 seven-year one.

5. The five-year spread goes down of 3 bps and the seven-year of 1 bp. What is the P&L of the trader who set up a strategy for a 15 MEUR nominal for a maturity of five years?

We have that:

$$\text{PNL} = A_5 \times (-0.03\%) \times DV_5 + A_7 \times (-0.01\%) \times DV_7$$

$$\text{PNL} = -15 \times (-0.03\%) \times 4.35 + 11.3 \times (-0.01\%) \times 5.76 = 13\text{kEUR}$$

Exercise 4: Forward CDS.

The 3-year CDS spread on a firm is 40 bp and its 7-year CDS spread is 75 bp. Compute the the spread of the contract that buys protection on A during the period starting in 3 years and ending in 7 years. We assume that the recovery rate is 40% and the risk free rate is 4%.

A portfolio strategy to replicate the asset we want to price would consist in:

- buying a seven-year protection;
- selling a three-year protection.

Thanks to the no arbitrage assumption, we know that we have the equality of both portfolios and thus:

$$JF_{3-7} = JF_{0-7} - JF_{0-3}$$

So we compute the value of JF_{0-3} and JF_{0-7}.

T	3	7
r	4%	4%
R	40%	40%
s	0.40%	0,70%
λ	0.67%	1.17%
$r+\lambda$	4.67%	5.17%
DV	2.80	5.87
JF	0.01	0.04

So recalling that:

$$JF_{x-y} = s_{x-y} \times \int_x^y e^{-(r+\lambda)t} \mathrm{d}t = s_{x-y} \times (DV_y - DV_x)$$

We have $s_{x-y} = \frac{JF_7 - JF_3}{DV_7 - DV_3} = \frac{0.03}{3.07} = 0.97\%$.