Tutorial 4

Portfolio Models and ABS

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Exercise 1: From granular homogeneous portfolio to non-homogeneous portfolio.

First, we consider a granular homogeneous pool, with internal risk parameters : PD $= \mathcal{N}(s)$, LGD and ρ for the correlation.

1. Compute the mean and the standard deviation of losses on the portfolio (Hint: consider a finite number, N, of assets first and then generalize).

2. For each $\alpha \in [0;1]$, compute the α loss quantile.

We need to find x such as:

$$\mathbb{P}(L \leq x) = \alpha$$

<u>First method:</u>

We have seen that:

$$L = LGD \cdot \Phi\left(\frac{s - \sqrt{\rho}F}{\sqrt{1-\rho}}\right) \tag{1}$$

Thus:

$$\begin{aligned}
\mathbb{P}(L \leq x) &= \mathbb{P}\left(LGD \cdot \Phi\left(\frac{s - \sqrt{\rho}F}{\sqrt{1-\rho}}\right) \leq x\right) \\
&= \mathbb{P}\left(F \geq \frac{s - \sqrt{1-\rho}\Phi^{-1}(\frac{x}{LGD})}{\rho}\right) \qquad (2) \\
&= \underbrace{\Phi\left(\frac{\sqrt{1-\rho}\Phi^{-1}\left(\frac{x}{LGD}\right) - s}{\sqrt{\rho}}\right)}_{\text{portfolio loss cdf}} \qquad (3)
\end{aligned}$$

(2) to (3) comes from the fact that F is a symmetrical variable $(1 - \Phi(x) = \Phi(-x))$.

Therefore:

$$\begin{aligned}
&\mathbb{P}(L \leq x) = \alpha \\
\iff &\Phi\left(\frac{\sqrt{1-\rho}\Phi^{-1}\left(\frac{x}{LGD}\right) - s}{\sqrt{\rho}}\right) = \alpha \\
\iff &x = LGD \cdot \Phi\left(\frac{s + \sqrt{\rho}\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right)
\end{aligned}$$

We then have:

$$q_L(\alpha) = LGD \cdot \Phi\left(\frac{s + \sqrt{\rho}\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right)$$

<u>Second method:</u>

We can notice that L is decreasing with F which is the only stochastic term in (1) and since the quantile function of F is Φ^{-1},therefore:

$$q_L(\alpha) = LGD \cdot \Phi\left(\frac{s - \sqrt{\rho}\Phi^{-1}(1-\alpha)}{\sqrt{1-\rho}}\right)$$

F is symmetrical therefore $\Phi^{-1}(1-\alpha) = -\Phi^{-1}(\alpha)$ and:

$$q_L(\alpha) = LGD \cdot \Phi\left(\frac{s + \sqrt{\rho}\Phi^{-1}(\alpha)}{\sqrt{1-\rho}}\right)$$

3. For a given threshold $l_0 <$ LGD, compute $\mathbb{E}[(L - l_0)^+]$.

4. We now relax the homogeneity assumption and consider that each asset has its own default probability asset i defaults as soon as its asset return $R_i = \sqrt{\rho}F + \sqrt{1-\rho}\epsilon_i < s + \sigma\bar{\epsilon}_i$, where all random variables $(F, (\epsilon_i)_i, (\bar{\epsilon}_i)_i)$ are normal, centered, reduced, independent variables.
Show that the loss distribution remains a Vasicek loss distribution, with modified parameters.

$$s' = \frac{s}{\sqrt{1+\sigma^2}}$$

$$\rho' = \frac{\rho}{1+\sigma^2}$$

Comment.

Exercise 2: Analysis of a securitization deal.

You are working in a bank and want to compute the required regulatory capital of a securitization deal. The deal consists in the securitization of a portfolio of mortgage loans which have the same size and the same maturity (7 years).

In this tutorial, we are going to use a simplified version of rating agencies methodologies. It consists in estimating for each loan a probability of default and an hypothesis in case of major changes in the market. These depend on the initial amount provided (1 - Loan To Value), the ratio Debt-to-Income, the geographical zone and data to assess a trend and volatility on the real estate market.

The tranching of the deal is as follows:

Class	**Amount**	**Owner**
A	900	Investors
B	70	Investors
C	30	Bank
TOTAL	**1000**	

We will use the following parameters:

- Probability of default for each borrower : 2 %;
- Loan to value: 75 %;
- Market-Value-Decline: 30 %;
- Repossession costs: 10 %.

1. What is the SFA capital on the pool before securitization?

2. A rating is given by agency ratings under the following conditions:

Number of time the expected loss is covered	Rating
1	BB
2	BBB
3	A
4	AA
5 or more	AAA

What are the ratings of the different tranches? What can you say about the methodology?

3. Propose another model to rate the tranches, assuming that you know the probabilities of default of all the corporate ratings.

4. Same question but assuming that there is an excess spread of 50 bp on the pool.

5. Knowing that the loans can be reimburse in advance, what is the effect on senior tranches?

6. Let us suppose that the senior tranche pays: EURIBOR(1y) + 30 bp and that the Asset Swap Spread is 10 bp (on 1 year).

Maturity	OAT	Bank
1 year	3.74 %	4.13 %
2 years	4.02 %	4.16 %
3 years	4.04 %	4.33 %
4 years	4.06 %	4.45 %
5 years	4.18 %	4.55 %
6 years	4.24 %	4.66 %
7 years	4.31 %	4.74 %

Rate	Last
EUR 1W EURIBOR	3.503 %
EUR 1M EURIBOR	3.625 %
EUR 2M EURIBOR	3.640 %
EUR 3M EURIBOR	3.662 %
EUR 4M EURIBOR	3.694 %
EUR 5M EURIBOR	3.717 %
EUR 6M EURIBOR	3.746 %
EUR 7M EURIBOR	3.768 %
EUR 8M EURIBOR	3.789 %
EUR 9M EURIBOR	3.802 %
EUR 10M EURIBOR	3.818 %
EUR 11M EURIBOR	3.831 %
EUR 1Y EURIBOR	3.841 %

Is the price of the senior tranche interesting? Which factors can explain this spread?