

# Tutorial 5

## Risk Modeling and Bank Steering

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An Excel version of the correction is available here: [http://defaultrisk.free.fr/tutorials/ENPC\\_CreditRisk\\_Lecture5\\_Tutorial.xlsx](http://defaultrisk.free.fr/tutorials/ENPC_CreditRisk_Lecture5_Tutorial.xlsx).

### Exercise 1: Going through the Credit Risk Weighted Assets formula.

We saw in class that the Credit Risk Weighted Assets formula is:

$$RWA = LGD \times \left( \Phi \left( \frac{\Phi^{-1}(PD) + \sqrt{\rho}\Phi^{-1}(0.999)}{\sqrt{1-\rho}} \right) - PD \right) \times MA \times SF \times MCR \times EAD$$

1. What does  $LGD \times PD$  represent?

|  $LGD \times PD$  represents the expected average loss in percentage.

2. We recall that the Vasicek model states that:

$$L | F \sim (1 - R)\Phi \left( \frac{\Phi^{-1}(PD) - \sqrt{\rho}F}{\sqrt{1-\rho}} \right)$$

with  $R$ , the recovery rate,  $\rho$  the correlation factor,  $F$  a normalized centered Gaussian parameter that can be considered as a systemic factor.

How can the following part of the Credit Risk Weighted Assets formula can be interpreted?

$$LGD \times \Phi \left( \frac{\Phi^{-1}(PD) + \sqrt{\rho}\Phi^{-1}(0.999)}{\sqrt{1-\rho}} \right)$$

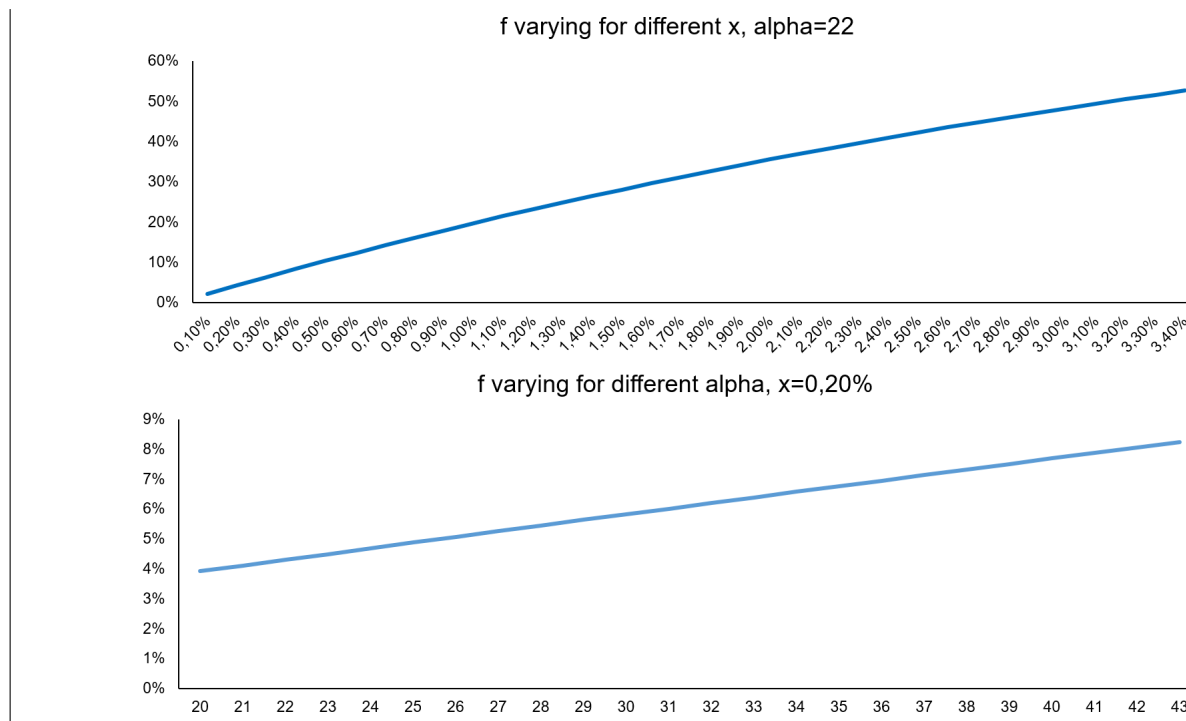
We know that:  $LGD = (1 - R)$ .

We also know that  $F$  is a systemic factor in the Vasicek model. We can match the Credit Risk Weighted Assets formula with the Vasicek model if we assume that  $F$  takes the value equal to the 0.001 quantile.

3. We recall that in the Internal Ratings-Based Approach (IRBA), the bank must estimate  $PD$ ,  $EAD$  and  $LGD$ , but not  $\rho$ . The correlation parameter  $\rho$  is indeed imposed by the regulator but its value depends on the counterparty of the studied contract:

Type	Value for $\rho$
Large Corporates Institutions	$0.24 - 0.12 \times \frac{1 - e^{-50 \times PD}}{1 - e^{-50}}$
Small and Medium Enterprises with turn over < 5 MEUR	$0.20 - 0.12 \times \frac{1 - e^{-50 \times PD}}{1 - e^{-50}}$
Small and Medium Enterprises	$0.24 - 0.12 \times \frac{1 - e^{-50 \times PD}}{1 - e^{-50}} - 0.04 \times \left( 1 - \frac{T-5}{45} \right)$
Residential Mortgages	0.15
Revolving	0.04
Other retail exposure	$0.16 - 0.13 \times \frac{1 - e^{-35 \times PD}}{1 - e^{-35}}$

a. Let us define  $f(\alpha, x) = \frac{1 - \exp(-\alpha x)}{1 - \exp(-\alpha)}$ : study how  $f(\alpha, x)$  evolves for any given value of  $(\alpha, x)$  in Excel.



b. In each case, show that the correlation is bounded and give the boundaries.

$f(\alpha, x)$  is in fact a weight equal to 0 when  $x = 0$  and to 1 when  $x \rightarrow \infty$ . Thus, the correlation is either fixed (Residential Mortgages, Revolving) or bounded (Large Corporates Institutions, Small and Medium Enterprises, Other retail exposure). For example, the correlation of Large Corporates Institutions is bounded by 0.12 and 0.24.

c. For an exact same contract, with different contractor types, in which cases are the Credit Risk Weighted Assets formula advantageous?

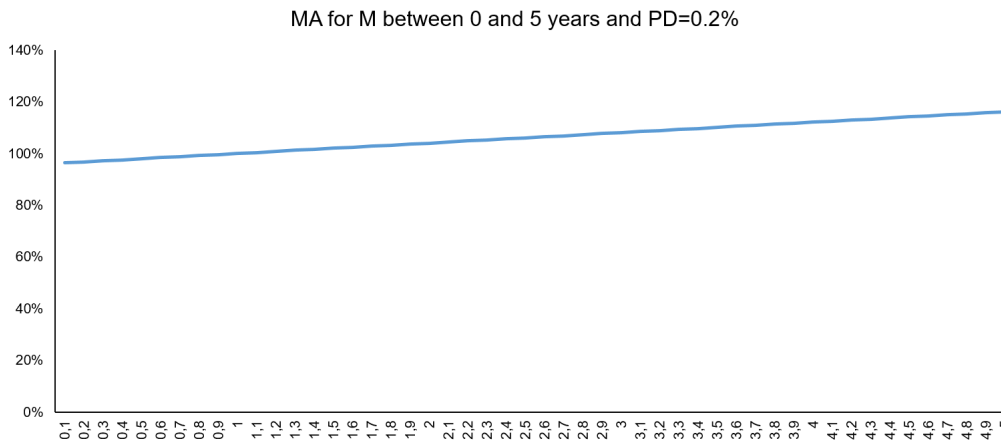
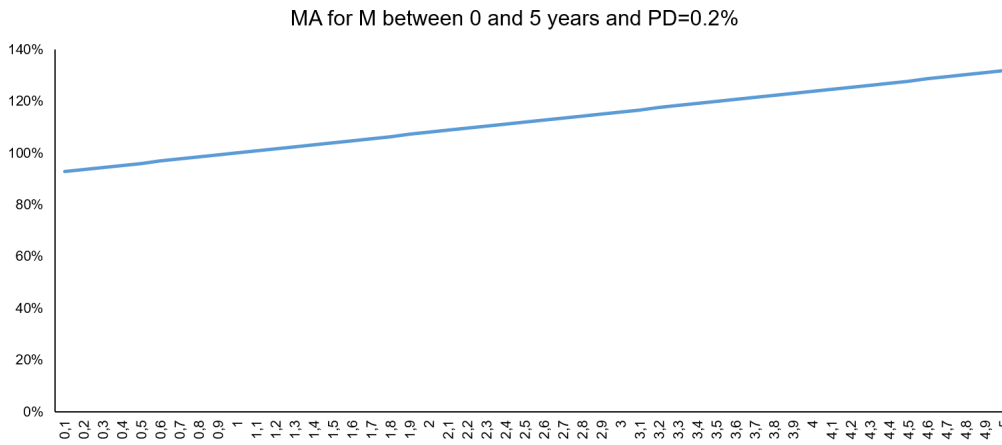
In general, we can see that retail banking activities with private clients are less penalized than investment banking for a same Probability of Default.

4. MA, the maturity adjustment is equal to:

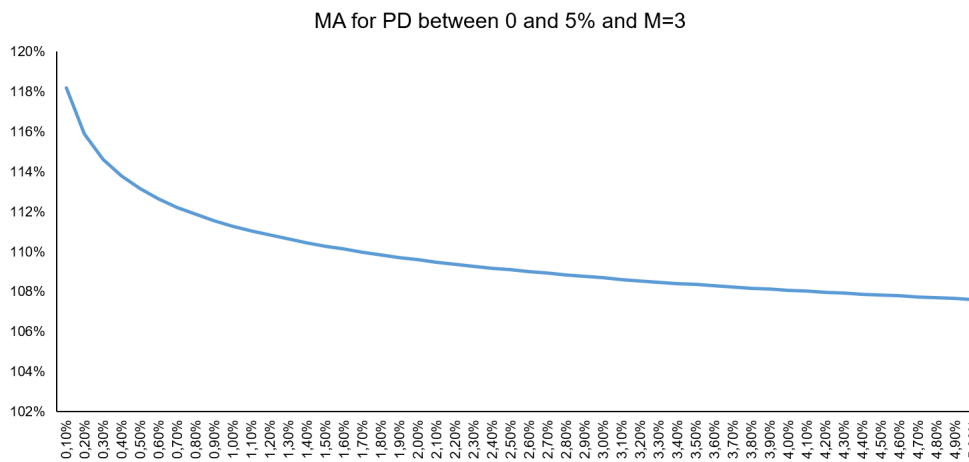
$$MA = \frac{1 + (M - 2.5) \times b}{1 - 1.5 \times b}$$

with  $b = (0.11852 - 0.05478 \times \log(PD))^2$  and  $M$  the maturity of the studied loan, bond, contract in general.

a. Plot MA for  $M$  between 0 and 5 years and  $PD = 0.2\%$  and  $PD = 4\%$ .



b. Plot MA for PD between 0 and 5 % and M = 3.



c. According to you, where does  $b$ 's formula come from?

$b$ 's expression might be the result of a regression based on economic data. In general, MA is a factor that is added in order to take into account the maturity as PDs are one-year PDs.

5.  $SF$  is called the Scaling Factor and is a regulatory coefficient equal to 1.06: what can you say about it?

$SF$  is a 6% add-on with no economic meaning. It is a margin of conservatism to capture, among others, the risk carried by the model.

6.  $MCR$  is the Minimal Capital Requirement and is equal to 12.5. According to you, where does this value come from?

In this exercise, we have computed the unexpected loss of a counterparty: indeed, in question 1. and 2. we saw that the Credit Risk Weighted Assets formula is the difference between the 0.999 quantile loss in the Vasicek model and the average loss, times some factors.

As seen in class, the logic behind the regulator's mind is to have enough capital to deal with unexpected losses. As a bank, regulators expect us to have the same amount of equity as the UL.

As regulators usually use Risk Weighted Assets in their analyzes, we usually translate these equity requirements in RWA. Historically, the Cooke ratio imposed banks to have 8% of their Risk Weighted Assets in equity. In other words, for any given UL, the corresponding RWA was:  $\frac{1}{8\%} \times UL = 12.5 \times UL$ . This is the reason for the Minimal Capital Requirements value. It has never changed even though the necessary percentage of RWA that a bank must hold in capital has changed.

## Exercise 2: The study of Simple Bank.

The purpose of this exercise is to compute the RWA, the ROE, RAROC, WACC, EVA 1, EVA 2, RARORAC 1, and RARORAC 2 of Simple Bank.

In the balance sheet of the bank, the liabilities are:

- the equity that amounts to 70 kEUR and for which the shareholders expect a 10 % return on their equity;
- the long term debt that represents 300 kEUR, and for which the interest rate is 0.8%;
- the deposits, that amount to 971.5 kEUR and do not bear interests.

In the balance sheet of the bank, the assets are:

Detail	Maturity	Nominal	Credit score	Recovery Rate	Interest Rate
Residential Mortgage to Ms Henry	15	157 000	1	75%	1.70%
Residential Mortgage to Mr Martin	12	245 000	3	67%	6%
Loan to Marinot Inc.	3	100 000	2	10%	1.20%
Loan to Total SA	4	800 000	1	80%	1.10%
Revolving to John BODIT	2	1 500	4	25%	10%
Consumer Loan to Sylvie BATIN	1.5	38 000	5	30%	11%

Last year, Marinot Inc.'s revenues were 19 MEUR.

1. What are the Risk Weighted Assets of Simple Bank? We will assume Simple Bank has no market exposure and an operational RWA equal to 1,9 kEUR.

We fill the table using exercise 1 formula and get the following:

Detail	RWA
Residential Mortgage to Ms Henry	35 255
Residential Mortgage to Ms Martin	362 005
Loan to SARL Marinot	123 525
Loan to Total SA	560 478
Revolving to John BODIT	3 072
Consumer Loan to Sylvie BATIN	-

We assume that the last line of the balance sheet has been switched in provisions and thus must not be taken into account.

In conclusion, the RWA of Simple Bank are equal to 1086 kEUR.

## 2. Is Simple Bank compliant with Basel III?

We will assume that Simple Bank need 10.5% of its RWA in capital, thus 114 kEUR. As it has only 70 kEUR of equity, Simple Bank is not compliant with Basel III.

## 3. The Net Income of Simple Bank is 2.1 kEUR, what is its Return On Equity?

We have seen in class that:

$$\text{ROE} = \frac{\text{Net Income}}{\text{Regulatory Capital allocated to the BL}}$$

Thus, the current ROE is  $\frac{2100}{70000} = 3\%$  and the one would Simple Bank be compliant with Basel 3,  $\frac{2100}{114055} = 1.8\%$ .

## 4. What is Simple Bank RAROC?

We recall that:

$$\text{RAROC} = \frac{\text{Expected net income} - \text{Expected Loss}}{\text{Economic capital}}$$

To get the expected net incomes we multiply the interest rates by the nominal. To get the expected losses, we multiply the EAD, by the PD, by the LGD. We get the following results:

Detail	Expected net income	Expected loss
Residential Mortgage to Ms Henry	2 669	78.5
Residential Mortgage to Ms Martin	14 700	3 234
Loan to SARL Marinot	1 200	720
Loan to Total SA	8 800	320
Revolving to John BODIT	150	202.5
Consumer Loan to Sylvie BATIN	4 180	26 600

In conclusion, the numerator of the RAROC is  $31699 - 31155 = 544$ . Thus, the RAROC is equal to  $\frac{544}{70000} = 0.8\%$ .

## 5. The purpose of this question is to take into account the cost of capital in the measurement of the return of Simple Bank.

### a. What is Simple Bank WACC?

We know that the liabilities are:

Type	Amount	Expected rates
Equity	70	10%
Long term debt	300	0.80%
Deposits	971.5	0%

The WACC is thus equal to  $\frac{70 \times 10\% + 300 \times 0.80\% + 971.5 \times 0\%}{70 + 300 + 971.5} = 0.7\%$ .

### b. What is Simple Bank EVA1 and EVA2?

We recall that:

$$\text{EVA} = \text{Expected net income} - \text{Expected Loss} - k \times \text{Economic Capital}$$

Thus,  $\text{EVA} = 31699 - 31155 - 0.7 \times 1341 \times 1000 = 534.6$  EUR. EVA2, using the regulatory capital would give a WACC of 1% and thus  $\text{EVA2} = 31699 - 31155 - 1 \times 1385 \times 1000 = 530.2$  EUR.

c. What is Simple Bank RARORAC1 and RARORAC2?

In the case of Simple Bank, RARORAC1 and RARORAC2 are the same as Simple Bank has not enough capital to be compliant with Basel III. The RARORAC is equal to the RAROC minus the WACC,  $RARORAC = 0.8\% - 0.7\% = 0.08\%$ . In conclusion, Simple Bank is really not a good investment for a stock investor.

The credit scores used by Simple Bank correspond to the following probabilities of default:

Credit Score	PD
1	0.20%
2	0.80%
3	4.00%
4	18.00%
5	100%