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Tutorial 6

Counterparty Risk

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Exercise 1: Computing counterparty risk on an interest rate swap.

We consider a discrete dynamic of interest rates. The date of computation is $t = 0$, and we suppose that the future states of the world are the ones of a binomial tree on four periods, i.e., $t = 0$, $t = 1$, $t = 2$, $t = 3$ and $t = 4$. We suppose that the discount rate is equal to 0 and that the probabilities of reaching the next branches on each knot are both equal to 50%.
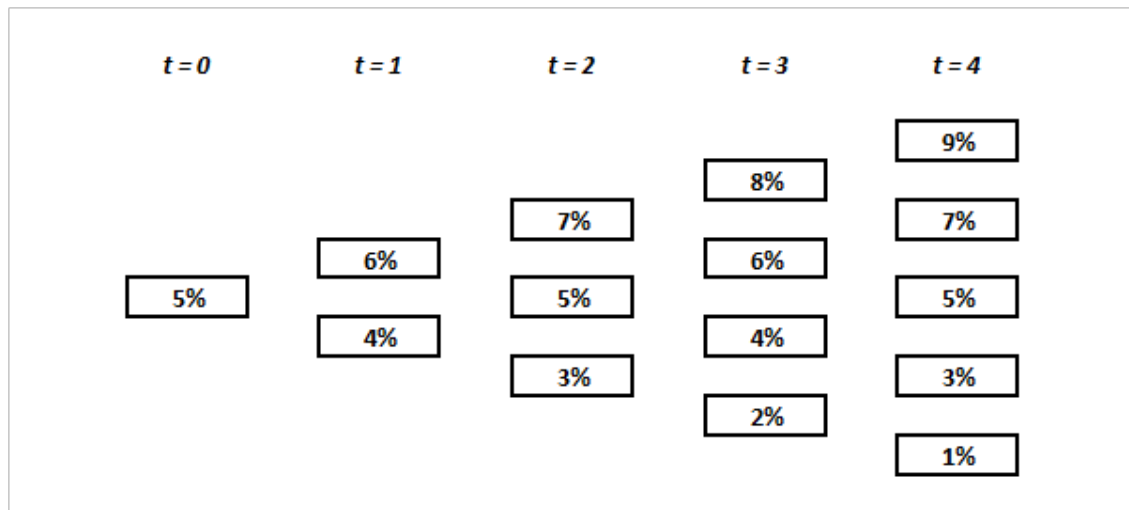
1. Fill the tree above with the cash-flows of a swap exchanging a fixed interest rate for a variable one; you pay the fixed rate at 5% and you receive the variable one on a notional of 100.

2. Deduce the market value of the swap at each date.

3. If the counterparty defaults on one of the knot, explain why your maximal credit risk is equal to the positive part of the market value of the swap at this date.

4. We consider we are in $t = 0$. For each date $t > 0$, compute the expectation of the positive part of the market value of the swap. We will call this curve EE(t).

5. Why can we say that the curve EE(t) corresponds to the future exposures that we have on the counterparty of the swap?

6. Let us now suppose that the (conditional) default probability of the counterparty at t, knowing that it had not failed at $t - 1$, is equal to 10%. Compute the survival probability at $t = 1$, $t = 2$, $t = 3$, and $t = 4$.

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7. What is the cumulative default probability between $t = 0$ and $t = 4$?

8. We suppose that the recovery rate is equal to 0. What is the expected credit loss on this swap on the whole life of the swap?

Exercise 2: Option pricing taking counterparty risk into account.

A company enters into a 1-year forward contract to sell 100 USD for 150 AUD. The contract is initially at the money. In other words, the forward exchange rate is 1.50. The 1-year dollar risk-free of interest is 5% per annum. The 1-year dollar rate at wich the counterparty can borrow is 6% per annum. The exchange rate volatility is 12% per annum.

1. Estimate the present value of the cost of defaults on the contract. Assume that defaults are recognized only at the end of the life of the contract.

2. Suppose now that the 6-month forward rate is also 1.50 and the 6-month dollar risk-free interest rate is 5% per annum. Suppose further that the 6-month dollar rate of interest is 5% per annum. Suppose further that the 6-month dollar rate of interest at which the counterparty can borrow is 5.5% per annum. Estimate the present value of the cost of defaults assuming that defaults can occur either at the 6-month point or at the 1-year point? (if a default occurs at the 6-month point, the company's potential loss is the market value of the contract.)