Tutorial 6

Counterparty Risk

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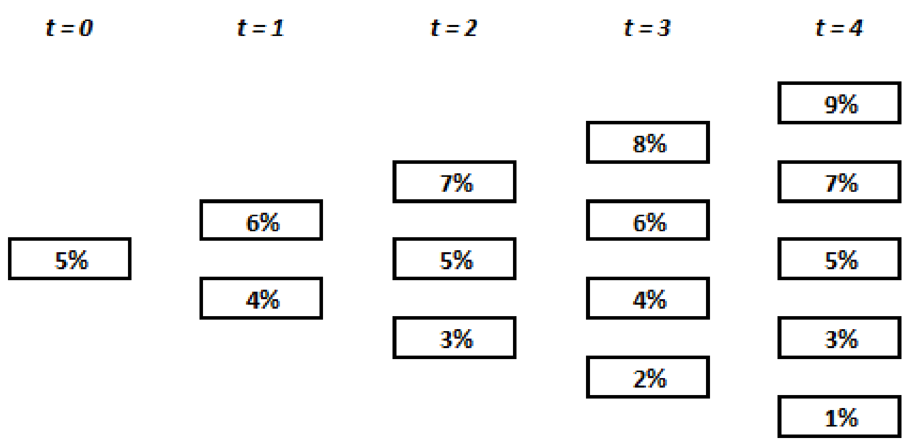
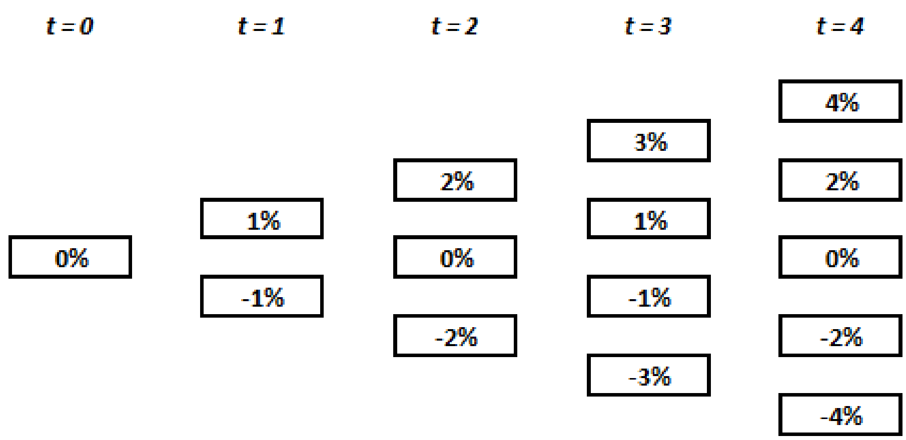
Exercise 1: Computing counterparty risk on an interest rate swap.

We consider a discrete dynamic of interest rates. The date of computation is $t = 0$, and we suppose that the future states of the world are the ones of a binomial tree on four periods, i.e., $t = 0$, $t = 1$, $t = 2$, $t = 3$ and $t = 4$. We suppose that the discount rate is equal to 0 and that the probabilities of reaching the next branches on each knot are both equal to 50%.



1. Fill the tree above with the cash-flows of a swap exchanging a fixed interest rate for a variable one; you pay the fixed rate at 5% and you receive the variable one on a notional of 100.

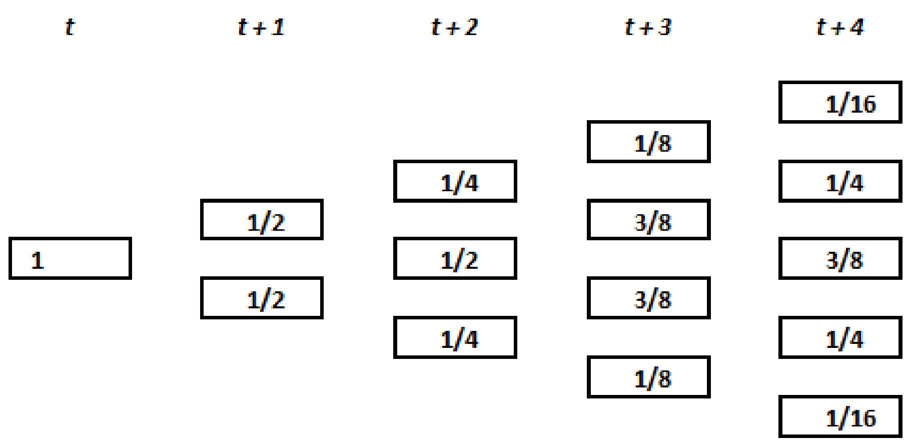
> We express the result in percentages by subtracting to the expected cash flow, 5%. The cash-flows are displayed in the tree below:



2. Deduce the market value of the swap at each date.

The market value on each knot is equal to the discounted expected cash flow at maturity ($t = 4$) of the swap.

The probabilities of the two branches coming from a knot being equal and equal to 0.5, the probabilities of each state for each subtree can be easily computed and summarized as follows:



For example, for $t = 0$, there is only one knot and the cash flow is equal to:

- 4% with a likelihood of 6.25%;
- 2% with a likelihood of 25%;
- 0% with a likelihood of 37.5%;
- -2% with a likelihood of 25%;
- -4% with a likelihood of 6.25%.

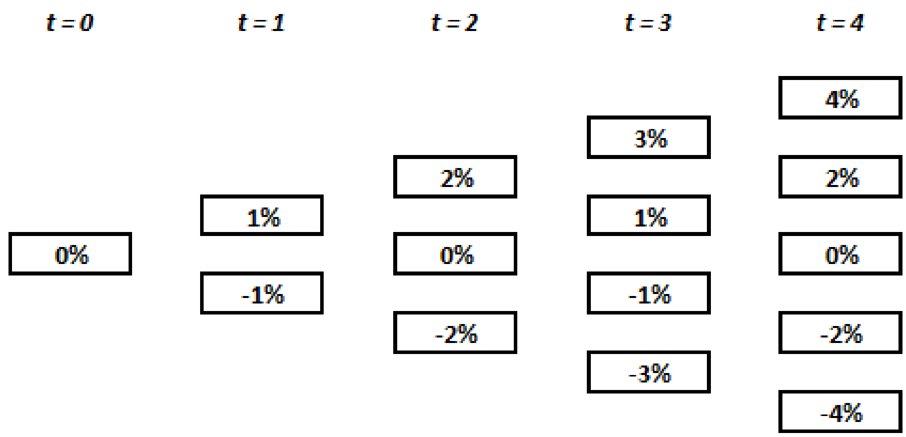
The discount rate being equal to zero, the discount expected cash flow is equal to 0.

In the same fashion the market value at time $t = 2$ on the top knot is give by computing the discount expected cash cash-flows. To do so, we use the subtree starting on the top knot at $t = 2$. The cash flows at maturity ($t = 4$) are as follows:

- 4% with a likelihood of 25%;
- 2% with a likelihood of 50%;
- 0% with a likelihood of 25%;

So the market value on this knot is equale to $4 \times 25\% + 2 \times 50\% + 0 \times 25\% = 2\%$ (expressed in %).

The other market values are displayed in the tree below:



Be careful ! The previous tree appears to be the same as the tree from question 1, but is not computed in the same way.

3. If the counterparty defaults on one of the knot, explain why your maximal credit risk is equal to the positive part of the market value of the swap at this date.

The credit risk born by the owner of the swap towards the swap originator is equal to its replacement cost if its value is positive, and zero, if not. Indeed, would its value be negative, it would mean that the owner owes money to the originator: in that case, the owner does not lose money in any case.

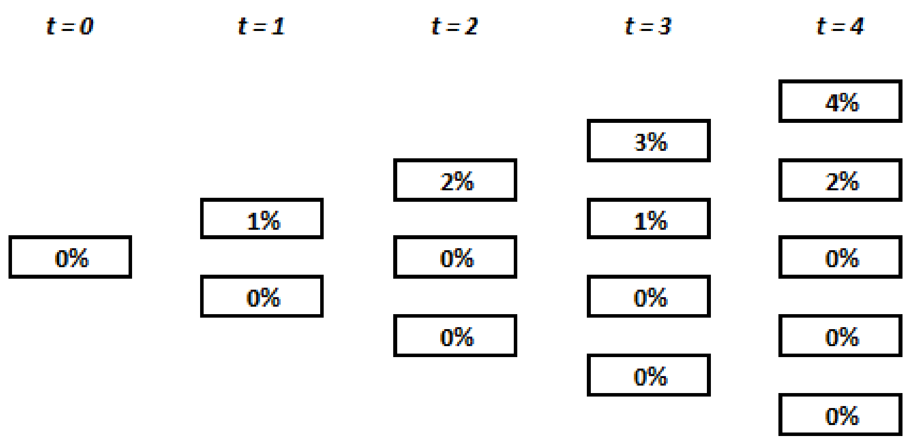
4. We consider we are in $t = 0$. For each date $t > 0$, compute the expectation of the positive part of the market value of the swap. We will call this curve EE(t).

The expected positive part of the market value of the swap, in each t is:

$$EE(t) = \sum_{s} PP(t,s) \times \mathbb{P}(s)$$

where, $PP(t,s)$ is the positive part in t in the state of the world s, and $\mathbb{P}(s)$ is the probability that the state of the world s happens.

The positive part of the market value is deduced from question 2 and given by the following tree :



Thus, we have;

- $EE(1) = 1\% \times \frac{1}{2} + 0\% \times \frac{1}{2} = 0.50\%$
- $EE(2) = 2\% \times \frac{1}{4} + 0\% \times \frac{1}{2} + 0\% \times \frac{1}{4} = 0.50\%$
- $EE(3) = 3\% \times \frac{1}{8} + 1\% \times \frac{3}{8} + 0\% \times \frac{3}{8} + 0\% \times \frac{1}{8} = 0.75\%$
- $EE(4) = 4\% \times \frac{1}{16} + 2\% \times \frac{1}{4} + 0\% \times \frac{3}{8} + 0\% \times \frac{1}{4} + 0\% \times \frac{1}{16} = 0.75\%$

5. Why can we say that the curve EE(t) corresponds to the future exposures that we have on the counterparty of the swap?

It corresponds to the expected credit risk exposure (positive part of the market value) born by the owner of the swap, towards the originator at each time period.

6. Let us now suppose that the (conditional) default probability of the counterparty at t, knowing that it had not failed at $t-1$, is equal to 10%. Compute the survival probability at $t = 1$, $t = 2$, $t = 3$, and $t = 4$.

The survival probability at $t = 1$ is equal to 90%. At $t = 2$, it is equal to $\mathbb{P}(\tau > 1) \times \mathbb{P}(\tau > 2 \mid \tau > 1) = 0.9 \times 0.9 =$ 81%.

Let $S(t)$ be the survival probability in t, we have:

- $S(0) = 100\%$;
- $S(1) = 90\%$
- $S(2) = 81\%$;
- $S(3) = 72.9\%$;
- $S(4) = 65.61\%$

7. What is the cumulative default probability between $t = 0$ and $t = 4$?

The cumulative default probability between $t = 0$ and $t = 4$ is equal to $1 - 0.6561 = 34.39\%$.

8. We suppose that the recovery rate is equal to 0. What is the expected credit loss on this swap on the whole life of the swap?

Let $P(t)$ denote the probability for the counterparty to default at time t. We have $\forall t > 0,\ P(t) = 0.9^{t-1} \times 0.1$ or using the survival probabilities computed in question 6, $\forall t > 0, P(t) = S(t-1) - S(t)$. The expected credit loss on this swap on the whole life of the swap is thus:

$$
\begin{aligned}
ECL &= \sum_{t=1}^{4} P(t) \times EE(t) \\
&= 10\% \times 0.50\% + 9\% \times 0.50\% + 8.1\% \times 0.75\% + 7.29\% \times 0.75\% \\
&= 0.2104\%
\end{aligned}
$$

Exercise 2: Option pricing taking counterparty risk into account.

A company enters into a 1-year forward contract to sell 100 USD for 150 AUD. The contract is initially at the money. In other words, the forward exchange rate is 1.50. The 1-year dollar risk-free of interest is 5% per annum. The 1-year dollar rate at wich the counterparty can borrow is 6% per annum. The exchange rate volatility is 12% per annum.

1. Estimate the present value of the cost of defaults on the contract. Assume that defaults are recognized only at the end of the life of the contract.

The cost of default can be written $u \times v$ with u the probability that the default occurs and v the value of the implicit option linked to the default.
Concerning u:
We have that:

$$u = 1 - \exp(-(0.06 - 0.05) \times 1) = 0.00995$$

Concerning v:
v is a call option as its pay-off is $\max(150 \times S_1 - 100, 0)$ where S_1 is the exchange rate in one year. More precisely, v has the value of 150 call options of pay-off: $\max(1 \times S_1 - 100/150, 0)$. Using the option pricing theory, we have that one of these calls is equal to:

$$(F \times \Phi(d_1) - K \times \Phi(d_2)) \times e^{-rT}$$

with:

$$d_1 = \frac{\log(F/K) + \sigma^2 T/2}{\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Here, we have, $F = K = 0,6667$, $\sigma = 12\%$, $T = 1$ and $r = 0.05$ and thus the option is valued 0.0303, and

$$v = 150 \times 0.0303 = 4.545$$

Eventually, the cost of default is:

$$4.545 \times 0.00995 = 0.04522$$

2. Suppose now that the 6-month forward rate is also 1.50 and the 6-month dollar risk-free interest rate is 5% per annum. Suppose further that the 6-month dollar rate of interest is 5% per annum. Suppose further that the 6-month dollar rate of interest at which the counterparty can borrow is 5.5% per annum. Estimate the present value of the cost of defaults assuming that defaults can occur either at the 6-month point or at the 1-year point? (if a default occurs at the 6-month point, the company's potential loss is the market value of the contract.)

In this case, the cost of default is equal to $u_1 \times v_1 + u_2 \times v_2$ with:

$$u_1 = 1 - \exp(-(0.055 - 0.05) \times 0.5) = 0.002497$$

$$u_2 = \exp(-(0.055 - 0.05) \times 0.5) - \exp(-(0.06 - 0.05) \times 1) = 0.007453$$

The value for v_1 and v_2 are 6-month and 1-year call options which value are 3.300 and 4.545.

The cost of default is thus:

$\mathrm{u}_1 \times v_1 + u_2 \times v_2 = 0.002497 \times 3.300 + 0.007453 \times 4.545 = 0.04211$